NRRI 87-17

A METHOD TO ESTIMATE LONG-RUN MARGINAL COST OF SWITCHING FOR BASIC TELEPHONE SERVICE CUSTOMERS

Clark A. Mount-Campbell
Associate Professor Industrial & Systems Engineering

Hisham M. Choueiki
Graduate Research Associate

THE NATIONAL REGULATORY RESEARCH INSTITUTE
1080 Carmack Road
Columbus, Ohio 43210

September 1987

EXECUTIVE SUMMARY

In this study we develop and examine a method to estimate a cost of "plain old telephone service" (POTS) that would be helpful in setting prices. POTS is defined to consist of an access line to a local network switch that will connect, on demand, the access line with other local access lines. In addition, POTS should provide the capability of connecting with a nationwide network switch, but the act of making that connection and the telephone call that would ensue is not part of POTS.

The main issues in the study are: What are the appropriate costs? Can the appropriate cost for POTS be separated from the costs for other services? What methods can be used to compute a cost figure for each part of the local network and aggregate them for the entire company? How can the cost figures be annualized?

Long-run marginal cost[1] is a most useful and appropriate item of information for setting prices. It is an important input for a number of pricing methods although not the only input. Demand information is also an important input to the pricing decision. The problem was also viewed from a decision theoretic point of view. In this alternative approach we examine the level of revenues that would provide adequate economic motivation to cause a telephone company to decide to add capacity. Such a method can derive a cost by applying engineering economic principles routinely used in the telephone industry for making capacity and configuration decisions.

The decision theoretic approach yields a cost that, <u>in theory</u>, is the same as a long-run marginal cost. <u>In reality</u>, because the equipment is available only in "lumps" which are added to existing facilities the decision theoretic approach results in an estimate of a constrained, average incremental cost and is considered here to be the superior method.

A telephone system consists of three distinct components: switching facilities, interswitch network facilities, and subscriber loop. The costs of expanding each of these facilities was assumed to be separably determinable with a total cost obtained by adding the individual component costs. While no empirical data exists to support or refute this assumption, the separability of the engineering function, and the separability of the activities for expanding these facilities suggest that the assumption is reasonable. However, the method of determining expansion costs must be

---

[1] Long-run marginal costs may be thought of as the current (not historical) cost of serving one additional customer where all resources are optimally varied to provide that service. If only a few inputs are varied then it is a short-run marginal cost.

uniquely developed for each facility type. In this report we developed a method for switching facilities and tested it in a pilot test with one actual switching machine.

A first goal of the method is to model the incremental costs. The model is then used to simulate those costs that a firm would compare with incremental revenues in order to economically justify the addition of new plant. The per-unit average incremental cost developed by the method will mathematically approximate a constrained long-run marginal cost. The constraint is that the modelled costs represent those for expanding existing facilities rather than those for constructing new facilities. Finally, an annual equivalent of the incremental cost is recommended so as to take tax effects and fill-rate forecasts into account and to move the one-time incremental cost closer to a rental price. The steps of the method are:

1. Select a sample of switches.

2. Establish an ESS 1A equivalent switch design for the non-ESS switches in the sample.

3. Have the company design expansions for the sample switches according to an experimental plan.

4. Organize the equipment lists for the expansion plans and examine for inconsistent patterns -- request revised designs when needed.

5. Fit a spline function regression model to four different sized expansions.

6. Compute an average annual unit cost for each output variable.

7. Compute an average annual cost per customer for residential and business customers.

The key step in the method is step 3. Rather than examine actual switch expansions where capacity to perform both POTS and non-POTS functions have been added, step 3 asks the telephone company engineers to use their computer aided design methods to design switch expansions according to an experimental plan that will allow separate estimation of the costs of the POTS functions. This approach unconfounds the costs of POTS and non-POTS functions that are generally confounded in data from actual expansions. The approach may be likened to performing controlled laboratory experiments. When an expansion is planned, it is described in terms of the following variables:

- access lines
- intraswitch busy-hour usage
- interswitch busy-hour usage
- DID trunks

The models in step 5 give expansion costs as a function of these variables. In the pilot study, interexchange busy-hour usage was also a variable. Its cost was found to be linearly separable from the other costs making it possible to hold it fixed in subsequent experiments. In some cases, DID trunk cost was not linearly separable from the other costs and was therefore retained as an experimental variable. The addition of a POTS customer is thought to affect only the first three variables listed above. Thus, given usage patterns for a POTS customer, it is known how the first three variables are affected by the addition to the system of POTS customers. Then from the cost models one can determine the construction costs of the added capacity.

Step 6 converts the construction costs to annual equivalent costs; taking into account income taxes, capital structure, depreciation rates, allowed rate of return, the rate at which customers are forecasted to fill the capacity, and (when available) operation and maintenance costs. The result is an annual cost for each of the variables listed above for each switch in the sample of switches selected in step 1. These costs per switch are then averaged across sample switches in such a way as to account for differential rates of growth in POTS customers at these offices.

Finally, step 7 computes the long-run marginal (or its practical equivalent--average incremental) cost for POTS customer based on their usage pattern.

A pilot study of the method was performed on a single Ohio Bell office. This pilot consisted of steps 2 through 5 and a part of 6 on that one office rather that a sample of several offices as prescribed in step 1. Steps 6 and 7 were demonstrated by simulating them using the results from the actual pilot office augmented with hypothetical (but plausable) data for two other offices to complete a sample of size 3. The result of applying steps 1 through 5 and part of 6 to the pilot switch was obtained under the following assumptions: capital structure is 40% debt with an average return of 9% on debt and a composit cost of capital of 13.2%; income tax rate was 46%; regulatory book life was 10 years and tax life was 5 years using an ACRS depreciation schedule. With these assumed values and actual cost values for the proposed expansion the optimal plan called for building three years worth of additional capacity at the beginning of year 1. This resulted in an annual cost of $3.24 per access line, $1.10 per intraswitch busy-hour CCS (centrum call seconds) and $22.18 per interswitch busy-hour CCS. These costs do not include operation and maintenance costs for the new facility nor do they include a fixed cost of $135,000 per switch expansion. This latter figure annualizes (under the above assumptions) to $23.13 per line. For a hypothetical POTS customer requiring 1 access line, 0.6 CCS of busy-hour intraswitch usage, and 1.3 CCS of busy-hour interswitch usage, the annual cost of that customer to the system would be $32.73, excluding the $23.13 fixed cost per line. If the fixed cost is allocated on the basis of lines, the total cost would be $55.86 of annual switching cost per customer.

While the method developed in this study does involve some expense for the telephone company in responding to step 3, it does not appear to be an excessive cost nor does it require an activity that is not ordinarily engaged in by telephone company engineers. It does provide data that are absent of confounding and allows the determination of costs for POTS separately from other services. Furthermore, it provides a cost figure representing the annual revenues needed for economic motivation to decide to expand facilities, and provides practical means to treat the problems of "lumpy" investment, forecasted growth rates, and expansions that are constrained by existing facilities.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

FOREWORD

This report was done under contract to the Public Utilities Commission of Ohio. PUCO has generously allowed us to publish it for general distribution. Its subject--determining the marginal cost of Plain Old Telephone Service--is timely and of wide interest.

Douglas N. Jones
Director, NRRI
Columbus, Ohio

December 30, 1987

ACKNOWLEDGEMENTS

The authors would like to express their appreciation to the Ohio Bell Telephone Company and especially the engineers and regulatory affairs staff members who gracefully accepted the onerous task of responding to our data requests.

In addition, we thank the Public Utilities Commission of Ohio for funding the project. It was invaluable having the personal interest and support of John Borrows and Diane Hockman for this project. It was through their efforts that information could pass from Ohio Bell to us. A debt is owed to Roger Montgomery who offered many helpful suggestions.

We are especially grateful to Carole Prutsman and Wendy Windle, who, in the eleventh hour, cheerfully bore the task of converting and correcting our word processor files to the NRRI system.

CHAPTER 1

INTRODUCTION

Definition of POTS (Plain Old Telephone Service)

The break-up of AT&T, the Computer I and II inquiries, and the Access Charge ruling are among the recent significant events in the evolution of national communications policy. These macro policy changes have been an enormous force for change in the way state governments can and should regulate the telephone companies in their jurisdictions. Additionally, technological advances in fiber optics, digital communications, and satellite communications have resulted in reduced costs for these technologies, making them competitive with, and in many cases superior to the older communications technologies that dominate the existing communications infrastructure in this country. These newer technologies also increase the variety of ancillary communication services that can be offered at reasonable prices. The main thrust of this evolving national policy has been to encourage competition in those markets where competition appears feasible because of technological advances.

One market that thus far seems to have been relatively unaffected by the changes in policy and technology is the market for "plain old telephone service" (POTS). POTS is normally thought to consist of an access line to a local network switch that will connect, on demand, the access line with other local access lines. Generally, the public would also expect POTS to provide the capability of connecting with a nationwide network switch, but the act of connecting to the nationwide network and the telephone call that would ensue is not part of POTS; instead, it is a part of the interexchange communication market that national policy intends to make competitive. This capability of connecting to a nationwide network is the part of POTS that causes us to refer to POTS as being "relatively" unchanged by policy and technology. The fact is, this part of POTS has been opened up to competition for large users of interexchange services. This competition is characterized by a user electing to connect to an interexchange carrier without using the POTS capability. When this happens the term "bypass" is applied.

It has been argued that the main reason bypass is economically feasible and, therefore, competitive with POTS, is that there is incorrect pricing of the interexchange calls made through POTS lines. Others argue that, even if interexchange calls made through POTS lines were priced correctly, some "bypass" would still occur. The reason is that a user may need to increase his capacity to make and receive interexchange calls, but does not need to increase his capacity to make or receive local calls. If the user's interexchange capacity is increased by adding POTS lines, then he would get and pay for an increase in capacity for both local and interexchange calling. In fact, some users may already have more POTS lines than needed for their local calls simply because they are needed for interexchange calls. Such users could also benefit from bypassing the local network.

All this brings into question whether it makes sense to include as a part of POTS the capability of connecting to a nationwide switch network. Our conclusion is that POTS should include this capability, primarily because as long as interexchange carriers have access to an exchange there is no cost to the POTS subscribers associated with having the capability of accessing them, but given the presently used telecommunications technology, there would be additional cost incurred if the telephone company had to provide a "local call only" type of POTS service. Of course, in the unlikely event that no interexchange carrier desired access to a particular exchange, then presumably the local telephone company would be obligated to invest in interexchange equipment in order to provide the "capability" to its POTS users. Such costs would be passed to the POTS users only if the use of the interexchange facility was not sufficient to cover its cost. Since this latter case would be a most unusual situation we prefer to discard it as a possibility. Thus, for purposes of this study, POTS, which is also the focus of the study, is defined to include the capability of connecting to a nationwide network, and that any competition that capability may have from "interexchange service only" types of bypass services is an irrelevant factor.

The Problem

The problem is that there are many services offered by a telephone company besides POTS. Many of these services are subject to competition whether or not they are actively regulated. It is also the case that the design of present-day central office equipment integrates into one machine many functions that are used in a variety of ways to offer both competitive services and POTS. If a commission is to move towards cost-based rates for POTS, the question is how can one determine the appropriate costs? The purpose of this study is to explore that question, to develop an approach for computing appropriate costs, and to assess the feasibility of the approach.

The main issues in this study are: What are the appropriate costs? Can the appropriate cost for POTS be separated from the costs for other services? What methods can be used to compute a cost figure for each part of the local network and aggregate them for the entire company? How can the cost figures be annualized? What are the pros and cons of the methods? While some of these issues are pursued, the main empirical work is focused on central office equipment (COE) of the electronic switching system (ESS) type. This represents a common and current technology, although telephone companies are beginning to move toward digital technology. As we examine the details of the issues we will find a number of additional subissues such as "lumpiness of investment," and how or whether to consider forecasts.

Organization of the Report

In chapter 2 there is a theoretical discussion of what costs might be appropriate and useful in setting rates for POTS services approached from two points of view. One is based on rudimentary economic theory, while the other is a decision theoretic viewpoint. Sufficient conditions for the equivalence of these two views of the problem are given. Chapter 3 presents a method for acquiring data and computing the types of costs defined in chapter 2. To illustrate the method, actual results from the empirical study of one Ohio Bell office are used. While this empirical work has been a major part of the study, the purpose is to derive a method to determine long-run marginal costs and it is in the context of the method itself that the empirical results are presented. In addition, there is a technical appendix that gives the empirical results. Chapter 4 contains conclusions and recommendations.

Appendix A contains a description of the analysis method used on data obtained in the pilot study of one Ohio Bell office. The data request for acquiring the Ohio Bell data is also found in appendix A. Appendix B is comprised of a data request form similar to the one used in the pilot study, but reduced to reflect a more efficient experimental plan than the one used in the pilot study. Appendix C presents the same sort of data request as appendix B but the experimental plan has been further reduced to an ultra-efficient form with respect to the amount of data it requests. Finally, as part of the project a Lotus spreadsheet program was developed to convert investment costs into annual equivalent costs. These annual equivalent costs include the effect of income taxes, probabilistic lives, equipment fill rates, and operation and maintenance expenses. Appendix D is a description of the program and instructions on its use.

CHAPTER 2

APPROPRIATE COSTS

Introduction

According to economic theory, a profit maximizing firm that sells its product in competitive markets will determine its production level such that the long-run marginal cost of one more unit of product is equal to its market price. If the same firm enjoys a monopoly in the market place then it will sell its product at its long-run marginal cost plus an adjustment that is based on the rate of change of the price with respect to a change in the quantity produced. In another approach, Baumol and Bradford have suggested the use of Ramsey prices for monopoly enterprises.[1] The Ramsey prices are selected to maximize the welfare of both producers and consumers. Like the monopoly prices, they are equal to adjusted long-run marginal costs. In this case, the adjustment is proportional to the inverse of the price elasticity for the product. The common factor in all these approaches is clearly long-run marginal cost. This does not mean that long-run marginal cost is the best cost figure on which to base prices, but is a useful cost to compute.

In the decision theoretic approach, we ask the question "what revenues would be sufficient to motivate the firm to add the capacity to produce additional units of a product?" Ignoring for the moment the uncertainty inherent in such a decision making situation, the answer to the question can be found in the engineering economy literature which is mainly concerned with the analysis of decision problems of this type.[2] One notes that the network and facilities planning activities in a telephone company are mainly

---

[1] Baumol, W.J., and D.F. Bradford, "Optimal Departures from Marginal Cost Pricing," American Economic Review 60 (June 1970), pp. 265-283.
[2] See, for example, Grant, E.L., W.G. Ireson, and K.S. Leavenworth, Principles of Engineering Economy, Seventh Edition, John Wiley & Sons, Inc., New York, 1982.

concerned with the analysis of decision problems of this type. If the present worth of the incremental revenues derived from the additional product is equal to or greater than the present worth of the minimum incremental cost of producing the additional product, the decision would be to add the capacity and increase production. In these problems the "cost of capital" is normally the interest rate used to discount the prospective cash flows to their present worth. Several different definitions for cost of capital can apply, but a composite cost of capital based on the weighted average cost of capital from all sources is the usual definition for a large, widely-held corporation. Instead of present worth, an annualizing calculation is sometimes used, and the comparison then is between the annual equivalent revenue increase and the annual equivalent cost.

Regardless which calculation is used to make the decision, if the increment of added capacity is one unit of product, then (under certain conditions which are discussed later) the decision criteria given above is really based on a long-run marginal cost. Since the calculations used in the decision theoretic approach include discounting the incremental cash flows at the cost of capital, it is easy to show that if the incremental product is priced below its long-run marginal cost, the present worth of the incremental revenues would be less than the present worth of the incremental costs, and the company would decide not to add the capacity. If the firm did decide to disregard its decision criteria and add the capacity anyway, the rate of return earned on the incremental investment would be less than the cost of the capital needed to finance the capacity expansion.

Such an exercise is incomplete, of course, in the case of a regulated utility. The regulated utility has an obligation to serve, and there are service standards that include maximum time intervals that subscribers should be made to wait before service is provided. Thus, if the service is priced below marginal cost and there is enough demand that additional capacity is needed, then to satisfy service standards the company is forced to either: (1) add the capacity and earn less than the cost of capital, or (2) find a short term way of satisfying the demand. In POTS service the short-term way to satisfy demand would be to connect the subscribers to the existing plant. This would cause its performance, as measured by blocked calls (or lost calls), to deteriorate. Eventually, the repeated application of the short-term solution would cause a decline in the quality of service.

Finally, enforcement of quality of service standards by a public service commission would force the company to add the needed capacity at a rate of return that is lower than the cost of capital.

If the cost of capital is the rate allowed by the regulatory commission and if the commission sets a price below the long-run marginal cost, then, in effect the commission will have contradicted itself. The contradiction comes from allowing a certain rate of return and then, through service standards, forcing the company to add capacity on which it will earn less than the prescribed rate of return.

Thus, the decision theoretic approach has appeal because it leads to a pricing principle without making assumptions about markets or about the profit maximizing objectives of the firm. That principle is that if a price stimulates enough demand to require additional capacity, the revenues generated from the demand at that price should be just sufficient to cause the company (while using standard engineering economy decision analysis techniques) to decide to add the capacity. Another way to express it is that the price makes it economically attractive for the company to meet service standards, thereby making it unnecessary to separately enforce the service standards in a long-run situation. Short-run enforcement of service standards is still needed to ensure that the timing of the company's long-run decision making does not unnecessarily inconvenience the subscriber, that is, to ensure that the company takes the short-run steps necessary to provide service until such time as the additional capacity becomes available.

Although the principle stated above can be supplied with assumptions that allow tracing the associated decision making back to a profit maximizing firm, the absence of such assumptions does not prevent us from being able to predict the economic behavior of the firm, whether or not it is able to maximize profits. In fact, prices would have to be very carefully set in order for the firm to maximize profits while following the principle. Even when prices are different from those that maximize profit, a firm is still better off following the principle.

Our conclusion is that long-run marginal cost is indeed a useful figure to compute, and in view of the decision theoretic approach and the typical service standards employed, the application of the above principle suggests that long-run marginal cost is also an appropriate quantity to compute for rate-making purposes.

Production Functions and Marginal Costs

In this section we summarize the relevant parts of the basic economic theory of the firm presented by Intrilligator,[3] and we relate this theory to the decision theoretic ideas mentioned earlier. Also discussed are the particular problems inherent in applying this theory to a real world situation.

A key to the economic theory of the firm is the production function which, in the single product firm, is the relationship between the maximum quantity that can be produced in a given period of time and the quantities of the factors of production that are inputs to the production process. The exact nature of the function is dependent on the technology employed by the industry in its production process. In its simplest form there are usually two factors of production considered--labor and capital. In its more complicated form the two factors can be disaggregated into many separate factors. Examples are dividing labor into unskilled labor, skilled labor, managerial labor, and dividing capital into land, production equipment, and transport equipment. For our purposes here we shall consider the simpler form of the function. Symbolically, the production function may be represented as follows:

$$q = f(x, y)$$

where q is the quantity produced, f is the production function, x is the amount of labor, and y the amount of capital input to the process. In the event the firm is a multi-product firm, the production function is a vector-valued function rather than a scalar-valued function, and q then becomes a vector. It is usually assumed that the production function is differentiable and that the derivative is continuous.

To develop the theory, it is supposed that a firm will adjust its output to some optimal level in the short-run by varying only one of the

---

[3] Intrilligator, Michael D., Mathematical Optimization and Economic Theory, Prentice-Hall, Englewood Cliffs, N.J. (1971), pp. 178-219.

inputs (usually labor, since it is easiest to vary in the short run). As we shall see, this idea will correspond to short-run marginal cost. In the long-run a firm is able to optimally adjust all its inputs in order to achieve an optimal production level. This situation will be seen to pertain to a long-run marginal cost.

Beginning the definitions of marginal costs, we assume that the firm does not have sufficient power in the supply side markets to affect the wage rates for labor or the "rental" rates for capital. Let p_x and p_y represent, respectively, the cost of one unit of labor for one period and the cost of one unit of capital for one period, and suppose that q is a given level of production, then a long-run cost function can be defined as follows:

$$C(q) = \underset{x,y}{\text{Minimum}} \{p\ x + p\ y \mid f(x, y) = q\} \tag{1}$$

and short-run cost functions can be defined as follows:

$$C_s(q,y) = \underset{x}{\text{Minimum}} \{px + py \mid f(x, y) = q\} \tag{2}$$

or,

$$C_s(q,x) = \underset{y}{\text{Minimum}} \{px + py \mid f(x, y) = q\} \tag{3}$$

Equation (2) is the more important of the two short-run cost functions since it minimizes only labor. This is generally what will happen in the short-run situation. The short-run cost functions are given only for completeness; our principal interest here is the long-run cost function given in (1) since it is the function from which long-run marginal costs are derived.

A marginal cost is the instantaneous rate of change in the cost of production with respect to a change in the level, q, of output. When that rate of change is computed using the cost function in (1), i.e., the long-run cost function, then it is a long-run marginal cost. Mathematically, the long-run marginal cost may be defined as follows:

$$MC_L(q) = \frac{dC(q)}{dq} \tag{4}$$

That is to say, it is the derivative of the long-run cost function.

Thus far, all the cost functions have depended on q which is as yet undetermined, but Intrilligator shows that if the product of the firm is sold in a competitive market at price p, then the firm will produce a quantity q that makes the marginal cost equal to p. This is shown by selecting a q that will optimize the following expression of profit:

$$\underset{q}{\text{Maximize}}\ pq - C(q) \tag{5}$$

To examine how this theory relates to the decision theoretic approach we first observe that if the q is a large enough value, then C(q+1) - C(q) is a close mathematical approximation to the derivative of the long-run cost function (i.e., it approximates the long-run marginal cost).[4] Second, we observe that C(q+1) - C(q) is the incremental cost of one more unit of production and p is the incremental revenue derived from that additional unit of production. According to the decision theoretic approach, if p is greater than or equal to C(q+1) - C(q), then the production should be expanded the one additional unit; and as long as the incremental revenue is greater or equal to the incremental cost, the capacity should be expanded until a point is reached that the incremental cost exceeds the incremental revenue. When this happens the last extra unit of production would not be added. Thus, we see that the decision theoretic approach yields the same solution as the classical economic theory.

In the case of a monopoly firm, the assumption is that the market price of the product is influenced by the quantity that the firm decides to produce. In such a situation the profit maximizing firm will choose a quantity q to produce such that the long-run marginal costs are equal to the marginal revenues.[5] Here again, the decision theoretic approach works to maximize profits if the incremental revenues are estimated to be something other than just p. In this case, the firm would consider the change in revenues resulting from a price change that would be necessary to stimulate demand for the incremental unit of production along with the revenues generated by the incremental unit. The total revenue change would be the

---

[4] Other approximations could be C(q) - C(q-1) or [C(q+1) - C(q-1)]/2.
[5] Marginal revenue is the derivative of the price-quantity function.

incremental revenues that would then be compared with the incremental cost in order to make the production capacity decision. The decision rule to expand or not to expand is exactly the same as before.

We conclude that while the basic economic theory defines the problem of the firm and specifies necessary conditions for maximizing profit, the decision theoretic approach developed in the engineering economy literature clearly applies the same concept in a mathematically approximate and realistic manner. We also note that in the decision theoretic view, C(q+1) -C(q) provides a useful and adequate definition of marginal cost, even though it is really an incremental cost. There remain a number of difficulties that must be resolved.

Not minor among the difficulties is the fact that in reality one does not measure C(q+1) - C(q) in order to decide whether or not to add capacity. The actual cost functions are highly constrained by the already existing plant so that what is measured is the cost of supplementing the existing plant, rather than the difference in cost of building two differently sized facilities. This difficulty is due more to the inadequacy of the economic theory discussed above (we deliberately chose a simple version of the theory) than it is a measurement problem. In these more complex and more realistic capacity determination problems, we find that the basic question is: "Does one build the extra capacity now, does one build it later, or does one not build it at all?" Here again, decision theoretic approaches are well suited to making such decisions and would optimally trade-off the cost of idle capacity built now with the present worth of incurring some extra fixed construction costs later caused by adding to an existing facility.

In considering the timing of the expansion decision, the difficulty is that the incremental costs and revenues that the decision would be based on are dependent upon a forecast of the need for service. The Long-Run Incremental Cost (LRIC) method developed by AT&T ran into difficulties at the Federal Communication Commission (FCC) for a number of reasons, and one of them was that it depended on forecasts. The view was that forecasts can be manipulated to show virtually any cost desired, thus the FCC rejected the

LRIC in favor of a fully distributed cost method.[6] While we have been unable to avoid the use of demand forecasts in the method presented in this paper, our rule was to keep costs and forecasts separated as much as possible. This allows us to clearly identify when and how the forecast has an effect on the costs.

Still another difficulty occurs when we recognize that most investment in telephone office equipment is "lumpy." By this we mean that the production function does not vary continuously, as is generally assumed in economic theory. Indeed, expensive items like central processing units, or line link networks each cause large jumps in capacity when they are added to a system. Thus, the production function that economic literature assumes to have continuous derivatives has instead discontinuous derivatives, and at perhaps a large number of points it has no derivative at all. Furthermore, many whole sections of the curve that represents the cost function will have a derivative of zero. These problems do not mean that the theory does not apply. They do mean, as is the case with all theory, that it serves as a guide and cannot be expected to lead directly to methods to estimate some real quantity.

A continuous derivative is not a needed assumption when using the decision theoretic approach, but again forecasting may play a role when we compute an incremental cost. Instead of letting the increment equal to only one unit of output as was suggested earlier, some of the lumpiness can be smoothed out by considering larger increments and then computing a per unit average incremental cost as an approximation. The problem is knowing how many units to average over. When considering revenues the problem is knowing at what rate revenues are produced. For example, in the first year it may be that only 50% of the added capacity is utilized, while the second and third years the figure may be 75% and 100%. This "fill rate," as we shall call it, is dependent on the forecasts of demand, while the revenue depends on the fill rate and incremental revenues per customer. Any decision to expand facilities must take the fill rate into account in order

---

[6] Revision to Tariff FCC No. 260 Private Line Service, Series 5000 (TELPAK), Memorandum Opinion and Order, 61 F.C.C. 2d 587 (1976).

to know whether the new facility will achieve revenue levels that would make adding the equipment the economically correct decision.

It should be pointed out that the economic theory and the decision theoretic approach discussed earlier have focused on the single product firm. The fact is, telephone companies are multi-product firms although this study is concerned with only one service--POTS. POTS itself may be viewed as at least three products according to our earlier discussion of the service. These are local access, toll access, and local busy-hour usage. In this study we have further separated local busy-hour usage into intraoffice local busy-hour usage and interoffice local busy-hour usage, since the capacities for these two parts of the usage service are mainly provided by different parts of the 1A switch. In any case, if we find that the structure of incremental costs is largely additive, then the separation of the incremental costs into pieces for each "subproduct" is possible. The major empirical work in this study was intended to identify and model the structure of the incremental central office equipment costs in order to determine if separation of costs is possible. That work is presented in the next chapter.

Summary and Conclusion

In this chapter we have discussed the usefulness of long-run marginal cost as an input to the pricing decision. The basic economic theory that was reviewed provided a clear definition of long-run marginal costs as well as a rationale for its role in establishing the behavior of the firm, given either competitive or monopoly markets for the firm's product. While the theory is presented in terms of a firm determining production levels, the relationship developed in the theory between price and long-run marginal cost offers clear guidance on setting prices. The presentation of the decision theoretic approach and its discussion seemed to more clearly connect prices with costs in that it demonstrated which relationship between the two would elicit the desired behavior of the firm with respect to whether or not it provides additional service to additional customers. It was also shown that under certain conditions (i.e., if incremental costs and incremental revenues are measured correctly in the decision theoretic approach), the decision theoretic approach is fully compatible with the economic theory. The decision theoretic approach may also overcome some of the potential difficulties that are present in the real world--difficulties that usually foil the direct application of certain basic economic theory.

Our fundamental conclusion is that the decision theoretic approach offers the greatest potential to be a practical tool for developing cost-based prices of POTS. The goal of the approach is to model the incremental costs. The model is then used to simulate those costs that a firm would compare with incremental revenues in order to economically justify the addition of new plant. The per-unit average incremental cost developed by the method will mathematically approximate a constrained long-run marginal cost. The constraint is that the modeled costs represent those for expanding existing facilities, rather than those for constructing new facilities. Finally, an annual equivalent of the incremental cost is recommended so as to take tax effects and fill-rate forecasts into account, and to move the one-time incremental costs closer to a rental price.

CHAPTER 3

A METHOD FOR ESTIMATING THE MARGINAL CAPITAL COST IN A CENTRAL OFFICE SWITCH

Introduction

Of particular interest is the potential the ideas presented in the previous chapter hold for the development of a practical method to estimate marginal costs in a real system. We begin with a general discussion of all the components of a total marginal cost of POTS. The discussion is then narrowed to the particular components of the marginal cost to which the study method applies. A method is then presented in a brief step-by-step form followed by a section giving the rationale for each step. Next is included a presentation and analysis of alternative means for accomplishing each step. Using data from Ohio Bell, the empirical results of an examination of the critical step in the method are presented through examples, and they appear to justify streamlining some of the more difficult procedures. In addition, a technical discussion of these empirical results may be found in appendix A.

Total Marginal Cost

For a typical customer POTS requires a number of physical components of equipment that when connected for all customers into an integrated system form a telecommunications network. At the system level, the individual components for the typical customer can be grouped into categories of equipment that are relatively independent with respect to their contribution to cost. For example, central office equipment (COE) and local distribution plant (loop) are relatively independent. A customer needs both COE and loop, and with present technology options in designing the loop plant, has little or no effect on how the COE would be designed or vice-versa.

In the very long-run, we could not claim this "relative independence" because one cannot assume technology will be fixed. However, by assuming a fixed technology an upper bound on cost is established, since there are only two legitimate reasons to introduce new technology. One is to provide the technical capability to jointly offer new services, and the other is to be able to provide existing services at lower cost. If the first of these two is the reason for introducing new technology, then it has nothing to do with POTS, and it would not be permissible to allow such new technology to increase the cost of POTS.

Other categories of plant are not so independent. An example of such a category is land and buildings which are clearly dependent on the sizing of COE and to some extent on the routing of loop. Land and building costs are also extremely lumpy and will most likely need to be spread, rather than treated in a pure marginal cost fashion.

To summarize, the categorization of both plant investment costs and expenses is shown in figure 3-1. We propose a practical method of determining a total marginal cost that addresses each block in the figure using an estimation method developed especially for that block. Thus, for example, one would determine a marginal cost for COE, a separate marginal cost for loop, one for interoffice outside plant, etc. Assuming these categories are relatively independent leads to a completely additive model for combining these costs into a total. If there are any categories where costs are not independent of others, a different model for combining them into the total will be needed. For now, it is assumed that the capital cost categories shown in figure 3-1 have an additive cost structure.

For the present study our attention has been focused almost exclusively on COE which is represented by the shaded block in figure 3-1. There are two reasons for focusing on COE. First, of all the system components it is one of the most difficult to estimate as to its contribution to the marginal cost of POTS. Therefore, it is the best area to focus research on the feasibility of the approach. Second, the amount of investment in the four major categories of imbedded plant owned by Ohio Bell in 1985 represented 41% of the total. The other percentages, as shown in fig. 3-1, are approximately 31% for loop, 15% for interoffice outside plant and 12% for land and buildings. If we assume that marginal costs will occur in

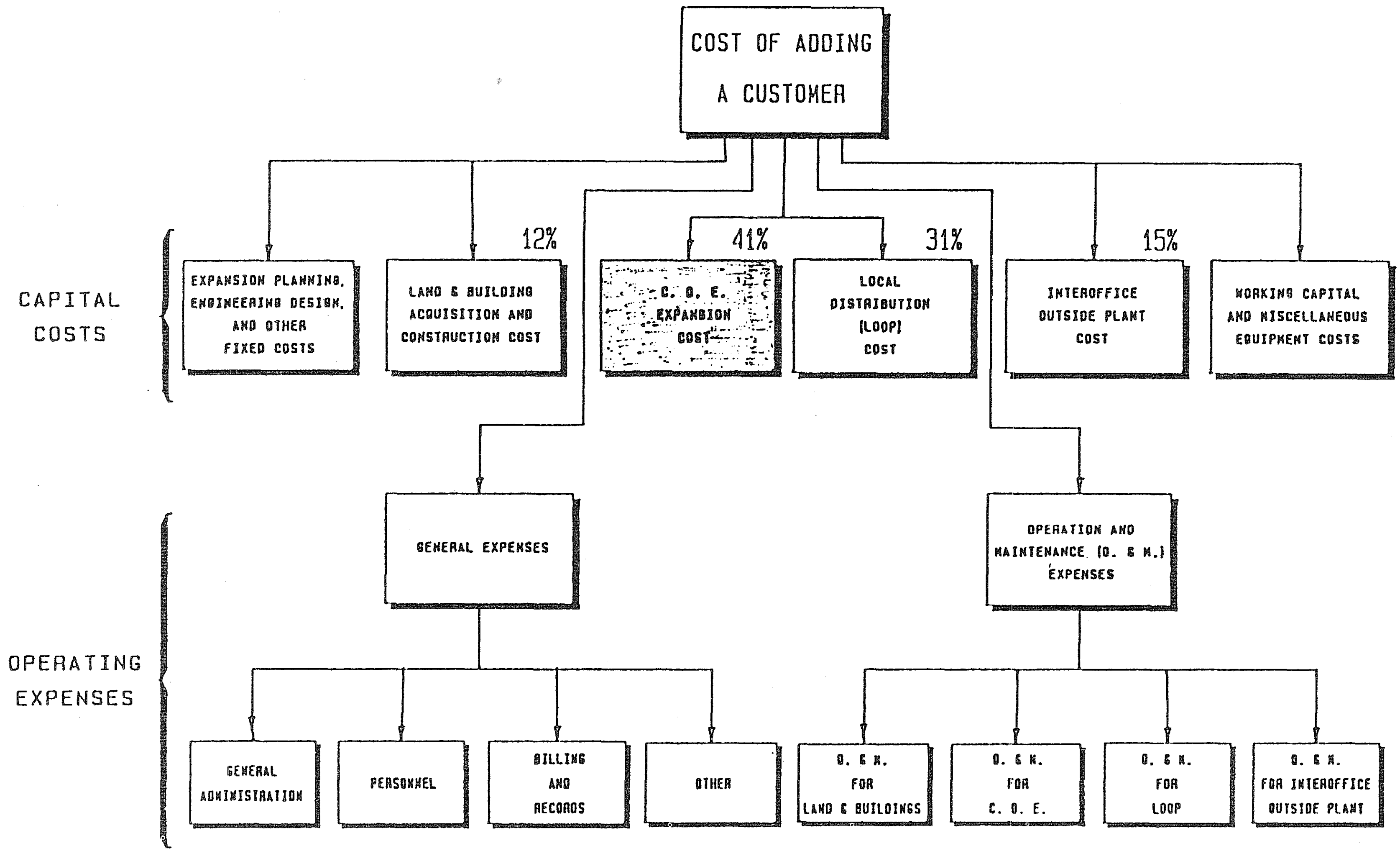

Fig. 3-1. Analysis of cost

approximately the same proportions as the present embedded costs, then COE is the largest piece of the total. The expansion planning, engineering design, and other fixed costs are not separately determinable and are generally accounted for as part of the investment in plant accounts.

The second largest piece of the total is loop and some attempts were made in this project to extend our work to loop. However, Ohio Bell has had an ongoing study of loop costs for well over a year and their representatives were reluctant to release the basic data gathered in their study until it is in a final form. As a result, we were unable to obtain any data relative to loop plant. However, should Ohio Bell's loop study prove useful for our purposes (and it appears that it will) then approximately 72% of the total capital costs will have been accounted for in the two studies. Thus, these two studies address the major parts of the problem as defined by dollar amounts.

We now present the general method for estimating the contribution of COE to the marginal cost of POTS customers.

Cost Study Method for COE

As was discussed in Chapter 1, the typical POTS customer requires an access line and imposes busy-hour traffic on the system. The busy-hour traffic is both intraswitch and interswitch if the community has more than one office. Even when a community has only one office there may be more than one switching machine in the office. The usual arrangement is to connect the machine with trunking much like separate offices would be so that a call from a customer on one switch to a customer on the other switch involves much of the same type of equipment as would a call across town. Hence, when we refer to interoffice traffic we also include interswitch traffic.

The two main customer classes that receive POTS service are business and residence classes. In our framework, the only significant differences between these two customer classes are the amounts of the two types of busy-hour traffic they impose on the system. We shall determine the incremental cost of adding a number of customers to a switch. With the decision theoretic framework, such a cost is the threshold value for deciding to invest in the expansion. However, the incremental cost is expressed as a function of the components of POTS (i.e., access lines and busy-hour traffic of both types), rather than as a function of the number of customers added. Procedures are given for then computing the customer cost.

One of the major difficulties with this approach, as mentioned in chapter 2, is the lumpiness of the investment. It was suggested there that this would require forecasting the fill rate of new equipment. We also suggest averaging results over a large number of switches as a means of minimizing the effect of lumpiness, as well as a means of achieving a company-wide figure.

From a theoretical point of view, an average marginal cost (or incremental cost) is not very useful. The reason is that both the economic theory approach and the decision theoretic approach are based on the idea of the firm making optimal decisions about its needed capacity. These decisions are made on a project-by-project basis, and if the service is priced at the average marginal cost, and if an expansion project is proposed in an above average cost switch, it would be rejected. However, customers are not charged differentially if they are connected to different switches. It is also the case that while a switch may be an above average cost switch to expand at a particular time, it very well may become a below average cost switch to expand a second time at a later date. In other words, there is both temporal averaging of costs of expansions to a given switch and an averaging of expansion costs across switches. It should be noted that normal econometric methods of estimating production functions also achieve average marginal costs. Thus, we believe that as a practical matter averaging is unavoidable.

The overall philosophy of our study method is as follows: An industry-wide marginal cost is not our goal, nor is it our goal to determine what the marginal cost should be for a given company. Our view is that in a given company's territory that company has the "franchise" to provide service, and it is their skill and expertise that determines what the service will cost. Thus, our approach is to use the company as an experimental apparatus on which we will conduct carefully designed and controlled "experiments" (or simulations). This consists of having the company engineers develop expansion plans to satisfy demand scenarios that we specify. These scenarios are specified in such a way that we are able to estimate the contribution of access lines, intraswitch busy-hour traffic, and interswitch busy-hour traffic to the cost of the expansion. We then annualize these costs to determine the annual incremental revenue that is needed to economically justify the implementation of the expansion plan. These we will call the marginal cost of the three components of POTS service. Pilot work with Ohio Bell indicates that these contributions are additive so that a marginal cost model for any particular customer is easily constructed once the usage pattern of that customer is known.

We conclude this section by presenting in step-by-step form a general outline of the method for experimenting with the company and determining the marginal cost. This is followed by a brief discussion of the rationale of these several steps in the method. Following the rationale, a section is presented in which we examine alternate ways of accomplishing these steps. Note that each step in the list below is followed by a short title in square brackets.

1. Select a sample of switches. [Select sample]

2. Establish an ESS 1A equivalent switch design for the non-ESS switches in the sample. [Standardize technology]

3. Have the company design expansions for the sample switches according to an experimental plan. [Design expansions]

4. Organize the equipment lists for the expansion plans and examine for inconsistent patterns -- request revised designs when needed. [Audit designs]

5. Fit a spline function regression model to four different sized expansions. [Analyze]

6. Compute an average annual unit cost for each output variable. [Average]

7. Compute an average annual cost per customer for residential and business customers. [Compute]

It should be noted that step 2 is an Ohio Bell specific task. If the company under study is not Ohio Bell, the technology to which all switches in the sample should be "standardized" is the predominant modern technology for that company. Step 3 also deserves comment in that, for Ohio Bell it is a relatively easy step to perform because of the availabililty of a computer-aided design package for its 1A ESS switch. It is assumed that other companies have similar design aides that will facilitate their performance of step 3.

Method Rationale

Before proceeding with details of the steps of the method, it is appropriate to discuss the rationale for the several steps of the method. Among the first five steps, steps 3 and 5 are key to the problem. These two steps have the goal of establishing, for each switch in the sample, a relationship between variables that define POTS customers' demands on the system and the cost of expanding an office in order to satisfy that demand. In our case here, a POTS customer, as defined in chapter 1, is represented by a vector of demands with components consisting of a line (x_1), interswitch busy-hour usage (x_2), and intraswitch busy-hour usage (x_3).

The non-operating cost of an expansion built to serve a POTS customer consists of all cash flows associated with expanding the switch. This includes one time expenses to cover removal and rearrangement of existing equipment items, and the installation of additional equipment items. Because these cash flows have components that receive different tax treatments we seek to establish the relationship mentioned above separately for capitalized and expensed items. We designate these two components as capital costs (y_c), and expansion related expenses or simply expenses (y_e). Total cash flow at time zero is designated y_t and and is related to its components by the relationship: $y_t = y_e + y_c$, where we have made the simplifying assumption that all expansion cash flows occur at one point in time designated as time zero.

Because of the relationship given immediately above, only two of the three cash flows y_t, y_e, and y_c need to be modeled in steps 3 and 5. For purposes of exposition, the ideal relationship established by steps three and five would take the form:

$$y_t = a_1 x_1 + a_2 x_2 + a_3 x_3 \qquad (1)$$

$$y_c = b_1 x_1 + b_2 x_2 + b_3 x_3 \qquad (2)$$

where a_i, and b_i are model parameters established by the method. The marginal cost of adding a customer with, for example, one line, one intraswitch busy-hour CCS of use, and three interswitch busy-hour CCS of use would be the vector product:

$$(a_1, a_2, a_3)\begin{pmatrix}1\\3\\1\end{pmatrix} = a_1 + 3a_2 + a_3 \qquad (3)$$

The vector product,

$$(b_1, b_2, b_3)\begin{pmatrix}1\\3\\1\end{pmatrix} = b_1 + 3b_2 + b_3 \qquad (4)$$

is needed only to transform the one-time lump-sum marginal cost in (3) into an annualized rental rate.

Unfortunately, one cannot expect actual cost structures to be as simple as (1) and (2). Instead, in a preliminary part of this study, it seemed that costs for the different components of POTS interacted with one another as well as with other non-POTS variables such as DID trunks (x_4, which also acts as a proxy for DID trunks). The results suggested the possibility of interaction of some POTS variables with interexchange busy-hour CCS (x_5) as well. Furthermore, there are fixed costs associated with expansions, and due to lumpiness of equipment, costs depend on the size of the expansion and the present state of the office being expanded. The effect of the size of an expansion on its cost is dealt with by developing four cost models rather than just one. Each cost model represents the costs of one of four differently sized expansions. The effect of the present state of a switch could be treated by constructing a cost model for every switch the company owns. This approach would be enormously expensive and time consuming. This is the reason a sample of switches is chosen in step 1 of the method. The sample should, of course, be representative of the population of all switches.

Interactive effects on costs and fixed costs require a more complex function to reflect the cost structure. The form these models now take is as follows:

$$y_{ts} = a_{0s} + a_{1s}\, x_{1s} + a_{2s}\, x_{2s} + \dots + a_{5s}\, x_{5s} + a_{12,s}\, x_{1s}\, x_{2s}$$
$$+ a_{13,s}\, x_{1s}\, x_{3x} + \dots + a_{15,s}\, x_{1s}\, x_{5s} + \dots$$
$$+ a_{45,s}\, x_{4s}\, x_{5s}$$
$$\text{for } l_{1s} \le x_{is} \le u_{is}\ ;\ i = 1, \dots, 5;\ \text{and } s = 1, \dots, 4 \qquad (5)$$

$$y_{cs} = b_{0s} + b_{1s}\, x_{1s} + b_{2s}\, x_{2s} + \dots + b_{5s}\, x_{5s} + b_{12,s}\, x_{is}\, x_{2s}$$
$$+ b_{13,s}\, x_{1s}\, x_{3x} + \dots + b_{15,s}\, x_{1s}\, x_{5s} + \dots$$
$$+ b_{45,s}\, x_{4s}\, x_{5s}$$
$$\text{for } l_{is} \le x_{is} \le u_{is}\ ;\ i = 1, \dots, 5;\ \text{and } s = 1, \dots, 4 \qquad (6)$$

where the subscript s indexes the model and variables for a size s expansion, and where a_{0s}, b_{0s} are intercept terms that represent fixed costs when s = 1. Terms such as $a_{13,s}$ and $b_{45,s}$ are the coefficients for two variable cross product terms. These models could each include up to 10 such crossproduct terms. Finally, l_{is} and u_{is} are lower and upper bounds giving the ranges of variable values over which the size s expansion is defined.

The important finding of our examination of the specific nature of functions (5) and (6) for the one switch studied in the pilot is that only four cross-product terms in all eight models and only one of the two fixed cost terms are different from zero. The four cross-product terms that were nonzero always involved x_{2s} (interswitch busy-hour CCS) and x_{4s} (DID trunks). Two of these terms were present in size 1 and size 3 versions of the total cost models (equations 5) while the other two were also size 1 and 3 versions of the capitalized cost model (equation 6). The fixed cost term was nonzero only in the total cost model, indicating that it results mainly from expense type items. Once the nonsignificant terms in these models are eliminated, they become nearly as simple as the ideal models given in equations (1) and (2). The lack of any cross-product terms that include x_5 (interexchange busy-hour CCS) implies an additive switching cost structure between POTS and toll services. This allows x_5 to be held fixed for the experimental plans as proposed for step 3.

The analysis procedure used to establish the models in step 5 was to treat the four size models as four segments of a spline function and estimate all parameters simultaneously with constrained least squares. This approach permits one to accurately model the observed data points while retaining a simple mathematical structure for each size range. In reality, these models should not be viewed as having some theoretically justifiable structure; instead, they may be viewed as models to interpolate at points between actual observations. These interpolation models fit the actual observations extremely well, which cannot generally be said for ordinary regression with a single given model form.

The audit step (step 4) is, of course, always necessary whenever data are received in response to a data request. An examination of a response for face validity will often reveal errors. In the case of this study, the balance of experimental design used in the data request provides a way to organize the data in order to greatly strengthen a face validity test.

Step 2, the calibration step, is needed to make the expansion planning process begin with a switch which conforms with the switch in the actual office. The conformance is with respect to the current status of demand and capacity at the sample switch. The conformance with respect to technology is not necessarily achieved unless the expansion planning process can be done on any variety of switch for all the experiments that are requested in step 3. At Ohio Bell, only the 1 and 1A ESS switches could have expansion plans developed and costs estimated by a computer aided design package. Without such a package the response to step 3 may impose unduly high study costs on the company. There is even some doubt that calibration of a non ESS switch is a practical step. If it must be avoided, step 1 may be modified by selecting the sample from only the switches with the qualifying technology.

Steps 6 and 7 use the cost models developed in steps 1 through 5 to compute an estimate of the company-wide annualized marginal cost of a POTS customer. Since the cost function of an expansion is dependent on its size, the first question is how large an expansion should be assumed. In one sense, the larger the better, since this allows averaging lumpiness of investment over a greater number of units, thereby mitigating its effect. However, building a large expansion may create excess capacity for a significant period of time (depending on growth rate) and thereby cause an overestimation of marginal costs. This is where demand forecasts have their direct effect on the estimate of marginal costs. Given these demand forecasts these steps seek to determine how large an expansion should be where the annual equivalent cost of the expansion is minimized on a per line basis. This should represent a reasonably good economic sizing of the switch expansion.

Thus, a cost model is determined for each office in the sample in steps 1 through 5 and averaged in step 6 using a weighted average calculation. These cost models are then converted to a per customer cost (depending on customer class) in a fashion similar to the idealized calculations shown in equations (3) and (4). The weighted average referred to above weights each cost model with the probability that a customer will join the corresponding office. This provides a means of recognizing that the actual costs incurred with growth in customers is very much a function of which offices actually need expansion, and not just a simple average of expansion costs.

The next section will discuss in detail each of the steps listed earlier. Many of the details are appropriate for Ohio Bell where computer aided design is routinely used. Also included are evaluations of alternative methods of accomplishing these steps.

Detail Analysis of Steps

Step 1: Select sample

All switching machines owned by the company within the Ohio jurisdiction should be available for possible selection into the sample. The purpose of the sample is to provide the objects on which an experimental plan will be implemented so as to obtain a company-wide estimate of the marginal costs of access lines and local usage. Two basic sampling methods are considered:

1) random sampling, and
2) stratified random sampling.

The first of these, random sampling, is the simplest and is performed by selecting switches one-by-one in such a way that each switch has an equally likely chance of being selected.

The second plan, stratified random sampling, divides the population of switches into groups called strata and allocates a certain portion of the entire sample to each stratum. Each within-stratum sample is then drawn randomly as described above. Stratified random sampling has a benefit when the switches can be grouped into strata in such a way that the uncontrollable factors which influence the cost of expansions occur in a relatively homogeneous fashion within each stratum. Since our empirical work involved only one switch, we can only speculate as to how one should stratify the Ohio Bell switches. The leading candidate criteria for defining strata would have to be whether or not the switch is found in a single switch exchange, and whether or not the population surrounding the switch is growing rapidly, slowly, not at all, or decreasing. Another reasonable candidate might be the number of access lines.

One can achieve a significant benefit from a stratified random sampling plan only if some estimate of the within-stratum variance is available. Such knowledge would permit an "optimal" allocation of the sample size to each stratum.

Step 2: Standardize technology

A random sample of switches is likely to contain switches with several different switching technologies. Some of those technologies may be quite outdated in that they are not the technology of choice in present day practice. Examples are step-by-step and crossbar. When such switches need more capacity a number of design alternatives may be considered that avoid increasing the investment in the older technology. The only case in which the older technology would be selected is when it is economically competitive with solutions using the new technology. We assume that is roughly equivalent (on a marginal cost basis) to replacing the switch with the newer technology and then expanding that new switch. Therefore, when a sample switch employs an outdated technology, this step asks the company engineers to design a replacement switch that would use the newer technology. This "hypothetical" switch should have the same service characteristics as the real switch, and it will replace the actual switch in the sample of switches that were selected in step 1.

Since the sample switches are the ones that will be used to estimate expansion costs, the disadvantage of the standardization process is that it minimizes the range variation likely to be found. The purpose of this standardization is to be able to take advantage of the computer aided design packages that are readily available for switches that use the ESS equipment. This greatly reduces the cost of the data and increases reliability and consistency of results. One might also argue that costs that are related to present day technology are the only costs relevant to present day decisions. An alternative to this standardization step would come from modifying step 1 so that the universe from which the sample is drawn would include only 1 and 1A ESS switches. This would obviate the need for step 2.

Step 3: Design expansions

The cost of expanding each switch in the sample must be investigated in much the same way as has been done in the pilot work of this project. The type of results to be obtained for each switch is depicted in figures 3-2 through 3-9. These figures give the results that were obtained in the study of one Ohio Bell switch. In that study there were five variables used to describe an expansion. These variables were: access lines, intraswitch busy-hour centrum call seconds (CCS), interswitch local busy-hour CCS, direct inward dial (DID) trunks, and interexchange busy-hour CCS. Four different sizes of expansion were considered, and within each size two levels of capacity additions in each of the above variables were considered. The four sizes are designated size 1 through size 4, and an analytical model was fit to the data from each size. The reason for including in the study the two variables that do not pertain to POTS, i.e., DID trunks and interexchange busy-hour CCS, was that we were concerned with the possibility of a joint effect on cost of these variables with those that do pertain to POTS. Such joint effects turned out to be virtually non-existent. The only joint effect of significant magnitude was a cost savings that occurred when both interswitch local busy-hour CCS and DID trunks are added to the switch simultaneously. This joint effect occurred in the size 1 and size 3 expansions but not the size 2 or size 4 expansions. As a general rule, one cannot predict which size expansions will exhibit this joint effect because it is dependent on the existing structure of the switch being expanded, as well as on the size of the expansion.

Figures 3-4, 3-5, 3-8 and 3-9 show the effect of the joint cost savings on both the total cost, capital cost and the average costs when the entire benefit of the joint cost savings is allocated to the interswitch local busy-hour CCS.[1]

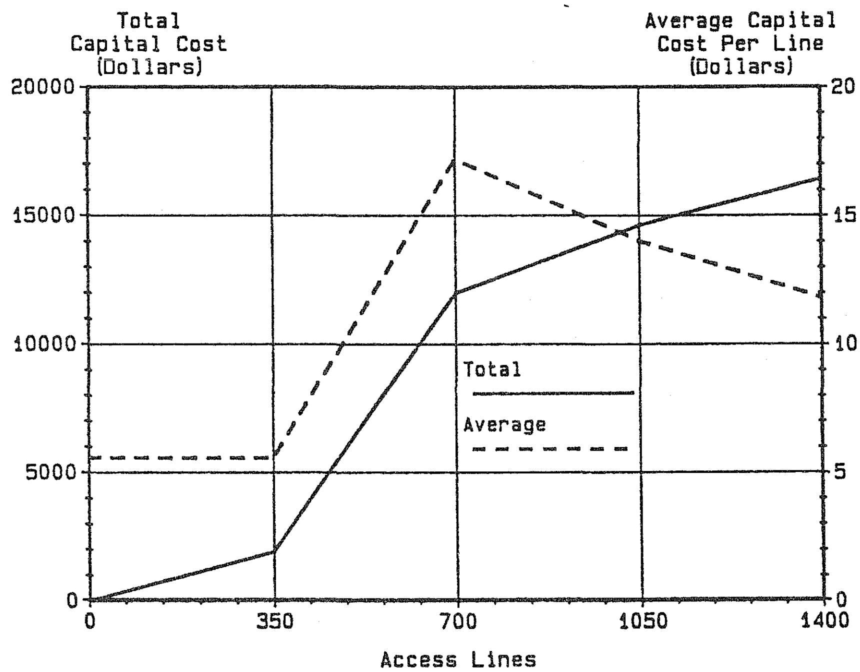

Fig. 3-2. Contribution of access lines to expansion costs

[1] The issue of how much should be allocated is more a pricing and/or policy question than it is a cost study question.

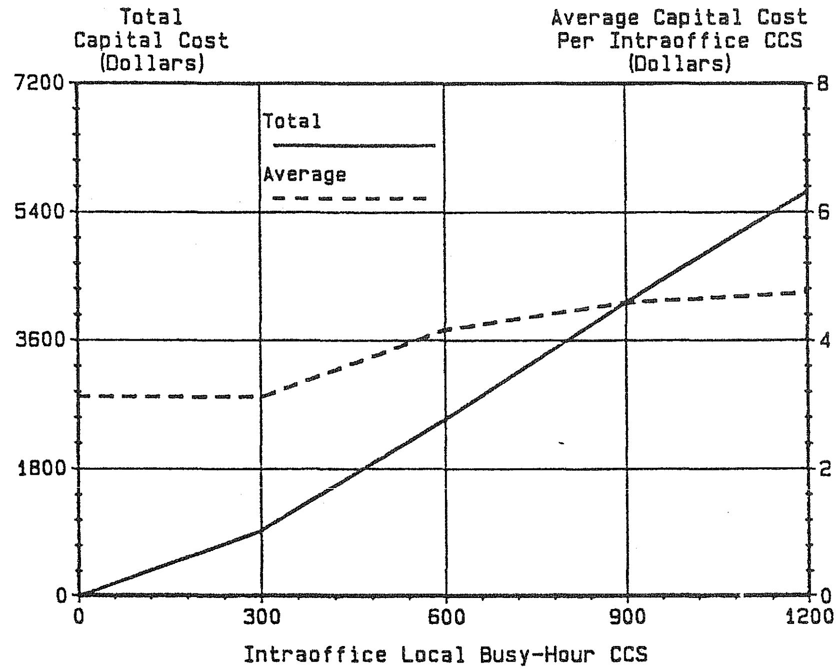

Fig. 3-3. Contribution of intraoffice traffic to expansion costs

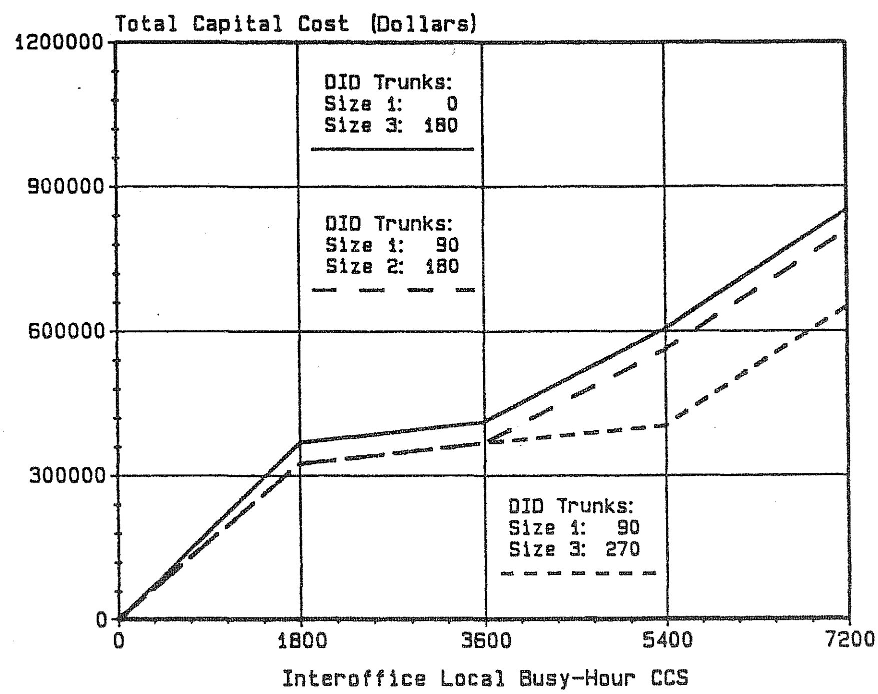

Fig. 3-4. Contribution of interoffice traffic to expansion costs

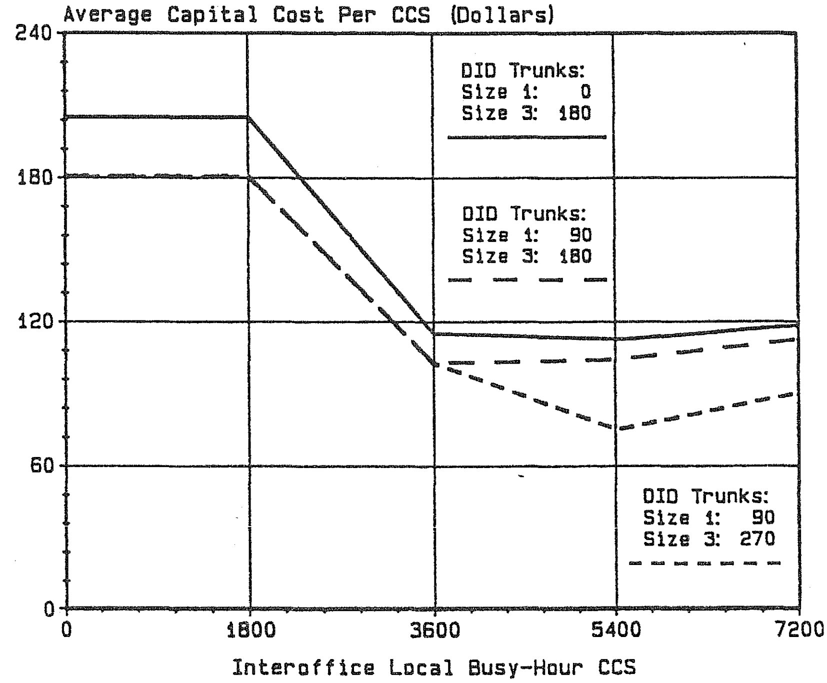

Fig. 3-5. Average interoffice traffic contribution to expansion costs

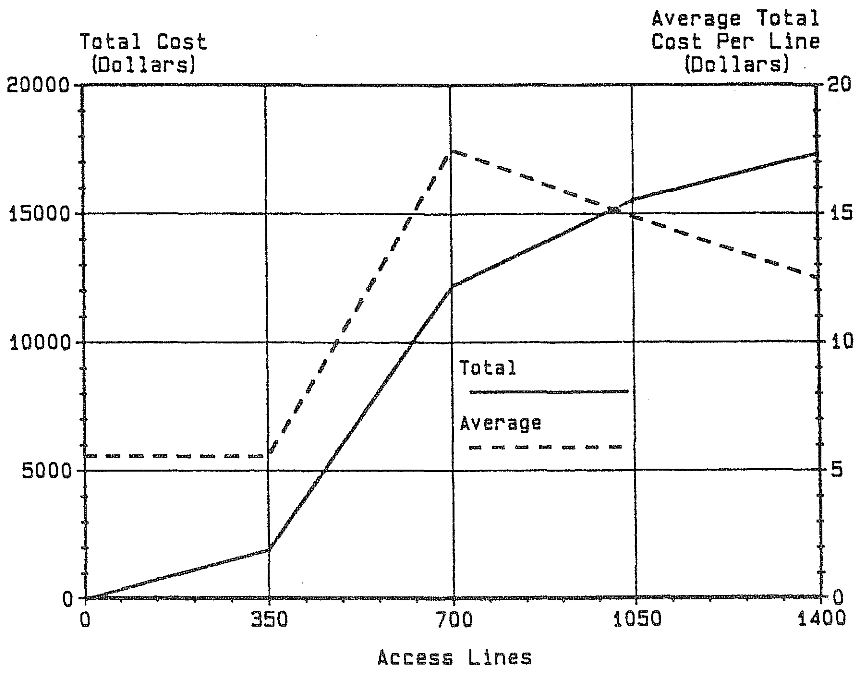

Fig. 3-6. Contribution of access lines to total expansion costs

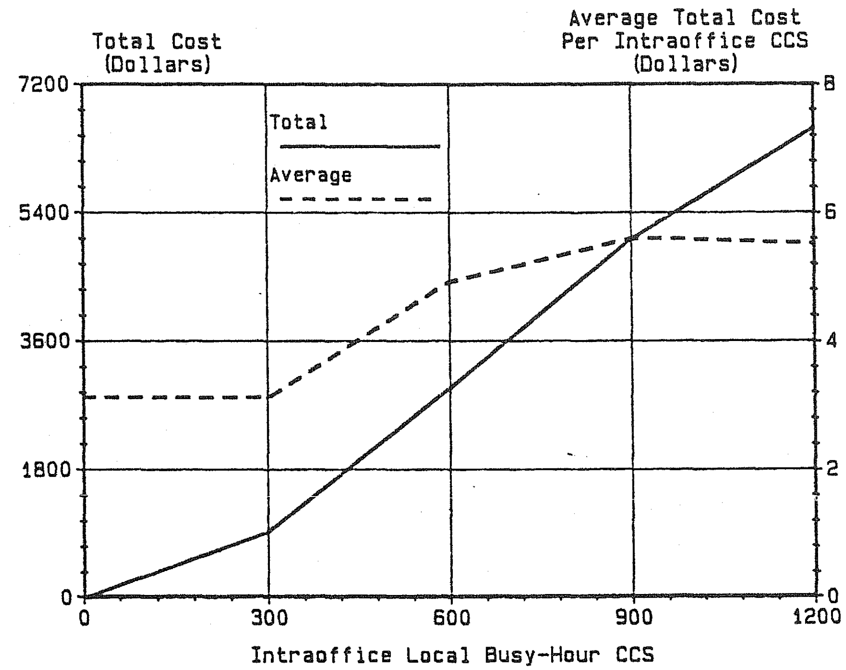

Fig. 3-7. Contribution of intraoffice traffic to total expansion costs

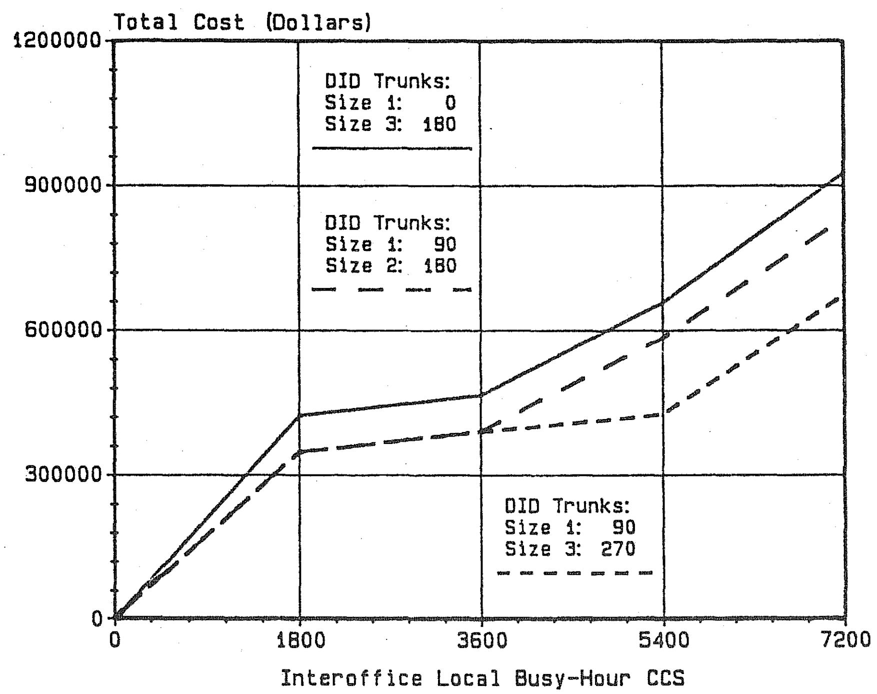

Fig. 3-8. Contribution of interoffice traffic to total expansion costs

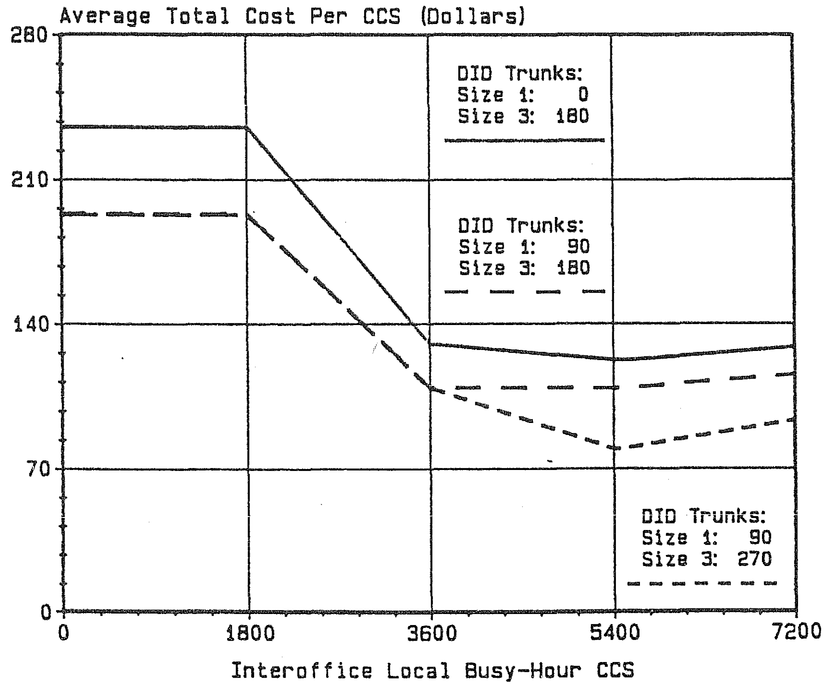

Fig. 3-9. Average interoffice traffic contribution to total expansion costs

The fact that most joint effects on costs are not present is a desirable result because it indicates that the cost structure is relatively simple. The simplicity is evident when one compares figures 3-2, 3-3, 3-6 and 3-7 with the other figures. This result also means that a simpler experimental design can be used when the method is applied to a sample of several switches. In the pilot, 16 experimental observations were necessary to examine the individual cost effects and the joint cost effects of the five variables. These 16 observations yielded 64 data points that were separated into 4 sets of 16 points each in order to establish a model of each of the 4 size expansions. Thus, 64 data points are actually necessary, but only 16 experimental runs are needed to obtain the 64 points because of the inherent features of the computer aided design package being used and because of our ability to take advantage of those features in the experimental plan.

When the method is applied to a sample of switches, the interexchange busy-hour CCS variable can be eliminated (held fixed at value zero) since it is not a POTS variable and since it has no interactive effect on cost with the POTS variables. Unfortunately, DID trunks cannot be similarly eliminated. However, elimination of the one variable reduces the number of data points needed to 32 from 64 and consequently reduces the number of experimental runs to 8 from 16. The resulting experimentation plan is given in appendix B in the format used in the pilot study. This is referred to as a 2^{4-1} fractional factorial design (each of the 4 sets of 8 points result in such a design.)

A second experimentation plan is given in appendix C. This plan achieves a further reduction in the number of experimental runs needed by essentially eliminating the DID trunk variable. In this case, we do not fully experiment with the DID trunk variable so that its effects cannot be measured, but we also do not hold it fixed to value zero. Instead, 45 DID trunks are added in all size 1 expansions, and that value is increased by 90 each time we shift to a new size. Since the DID trunk variable is not varied within each size, the number of required data points is again reduced to 16 from 32, and the number of runs needed to obtain those points is reduced from 8 to 4. The effect of this approach is to pass all of the savings that result from the joint addition of DID trunks and interswitch busy-hour traffic on to the interswitch busy-hour traffic variable. While this seems inequitable, it must be remembered that, if in reality many more DID trunks are added than were assumed in the experimental plan, more actual savings would occur than would be accounted for. The model would allocate all the accounted for savings to interswitch busy-hour traffic and those savings not accounted for would be left to the DID trunks. Thus, a form of sharing would take place.

Our recommendation is that in initial studies the 8 experimental run plan be used so that a more precise picture of the interaction between DID trunks and the interswitch traffic can be drawn. This will allow a procedure to allocate the cost savings according to a reasonable principle, rather than being hidden in the mathematics of a model. Once it is learned how to make the mathematics of the model accomplish the same allocation that would be made if the principle was applied, one could use the experimental plan in appendix B and reduce the data requirements and the cost of the study method.

Step 4: Audit designs

Once the telephone company engineers have designed the expansions for the sample switches, it is necessary to examine the results for errors or inconsistencies. While we cannot expect to be able to discover all errors, we can discover results that do not appear to make sense. This type of reasonableness check is facilitated by the fractional factorial experimental plan that defined the expansions to be examined. These plans are known for balance and symmetry so that when the data are systematically organized in a particular format, one can examine for balance and symmetry in the expansion designs developed by the company. When there is imbalance and asymmetry noted, the company can be asked for an explanation.

In the pilot study this process was very effective and proved to be an essential step in the acquisition of reliable data. The results of the pilot study will be used to illustrate the audit process.

In factorial experiments there is an ordering of the observations that is referred to as the natural ordering. Arranging fractional factorial observations into a natural ordering is a bit more complicated but is always possible.[2] The designs we recommend consist of four different sized expansions, with each size incorporating a series of expansions in which each variable assumes one of two values--called a high level and a low level. An observation within a given size expansion is then designated with a lettering system employing lower case letters to represent each particular variable. For example, in our pilot work "a" stood for access lines, "b" for DID trunks, "c" for intraswitch busy-hour CCS, "d" was interswitch local busy-hour CCS, and "e" represented interexchange busy-hour CCS. The lettering system referring to a particular combination of variables that corresponds to an observation uses a letter to represent each variable that is at its high level, and excludes letters for variables that are at their low level in the observation. Thus, for example, "a" refers to an observation in which variable "a" is set to its high level and all other variables are low, and "abe" denotes an observation in which variables "a,"

---

[2] See for example, Montgomery, D.C., and E.A. Peck, Introduction to Linear Regression Analysis, John Wiley & Sons, 1982.

"b," and "e" are at their high levels while "c" and "d" are at their low levels. (1) is used to denote an observation in which all variables are at their low levels. Because of the experimental plan specified for each size expansion, the all high observation, abcde, for one size is exactly the same observation as the all low observation, (1), for the next larger size.

The natural ordering for a full factorial involving the five variables listed above is as follows: (1), a, b, ab, c, ac, bc, abc, d, ad, bd, abd, cd, acd, bcd, abcd, e, ae, be, abe, ce, ace, bce, abce, de, ade, bde, abde, cde, acde, bcde, abcde.

Our experimental plan was a one-half fraction of the above plan and has the natural ordering as follows: e, a, b, abe, c, ace, bce, abc, d, ade, bde, abd, cde, acd, bcd, abcde. In this one-half fraction, the all low observation is not part of the plan but the all high is included. Even though the all low is not part of the design for a given size, it is available as the all high observation in the smaller size, since adjacent size expansions have the all low and all high observations in common. The size 1 expansion has, by default, the all low observation available as well, since it represents no expansion at all and, therefore, has no cost and no equipment added.

It is this natural ordering that is the systematic format needed to examine the data for errors. Tables 3-1 through 3-4 show, respectively, a matrix of equipment numbers for the size 1 through size 4 expansions. Each column represents a particular expansion plan described in terms of the five variables listed at the top of the column. The first five rows of these tables give the actual amounts of increased capacity in the expansion. The sixth line gives the factorial letter codes for the observation (expansion). Note there are a total of 16 expansions for each size and all are displayed in the corresponding table in the natural order moving from left to right. The main bodies of these tables contain the number of items of equipment, by type, that would be deleted or added in an expansion. The bottom rows of the tables give the capital costs and the total costs of making the various expansions in thousands of dollars.

There are very recognizable patterns in the placement and magnitudes of the numbers in the main body of table 3-1. Similar patterns are found in the main bodies of tables 3-2 through 3-4, but they are much harder to recognize. For that reason, tables 3-5 through 3-7 were created by subtracting the amounts of equipment needed for the all low expansions for a given size from the equipment lists of all expansions in the size. These new tables for sizes 2, 3, and 4 now appear very similar to table 3-1 for size 1, where the patterns in the numbers of equipment are more easily recognized. The audit process consists of examining for breaks in the patterns, or unexplainable shifts in the patterns. The two cost rows are included in this examination. To consider a specific case, table 3-8 has been constructed to simulate several of the situations encountered during the pilot study. Some of these situations required new expansion plans from the telephone company. Table-to-table comparisons should also be made. In table 3-8, areas of the table have been circled where questions of validity of the data should be raised.

In addition, an examination of the costs in table 3-8, which are actually incremental costs between size 2 and size 3 expansions, reveals an inexplicable shift in cost. For example, the "e" expansion in table 3-8 differs from the "abcde" expansion in size 2 by only a few more small items of equipment and yet the costs are significantly higher. It was discovered that errors had been made in several size 3 expansions. Comparisons across tables 3-1, 3-5 to 3-6 show a jump in costs when interswitch busy-hour CCS capacity is added in size 1 and size 4 expansions but not size 2 and size 3 expansions. This particular result was not an error and could be explained by the addition of a very "lumpy" piece of equipment in the size 1 and size 4 expansions.[3] That equipment was a line-link network (LLN). A small LLN was first added in the size 1 expansion with no change in the size 2 and size 3 expansions, but it was replaced with a larger LLN in the size 4 expansion.

Finally, we found that an electronic spreadsheet, such as Lotus 1-2-3, was an excellent tool to organize the data as described above. Other techniques were tried, such as numerical analysis of the equipment lists, but were found to be no more effective than the simpler pattern-recognition technique.

---

[3] In the study we were given the description of the equipment added under a protective agreement. In the tables we have expunged the equipment names to prevent revealing what Ohio Bell considers to be proprietary data.

TABLE 3-1

EQUIPMENT REQUIRED FOR SIZE 1 EXPANSION

SINGLE LINES ADDED		0	350	0	350	0	350	0	350	0	350	0	350	0	350	0	350	SINGLE LINES ADDED
DID TRUNKS ADDED		0	0	90	90	0	0	90	90	0	0	90	90	0	0	90	90	DID TRUNKS ADDED
INTRA USAGE ADDED		0	0	0	0	300	300	300	300	0	0	0	0	300	300	300	300	INTRA USAGE ADDED
INTER LOCAL ADDED		0	0	0	0	0	0	0	0	1800	1800	1800	1800	1800	1800	1800	1800	INTER LOCAL ADDED
INTER EXCHG ADDED		400	0	0	400	0	400	400	0	0	400	400	0	400	0	0	400	INTER EXCHG ADDED
DESCRIPTION	PRESENT	e	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3									1	1	1	1	1	1	1	1	EQU 1
EQU 2	2									1	1	1	1	1	1	1	1	EQU 2
EQU 3	0									2	2	2	2	2	2	2	2	EQU 3
EQU 4	1									1	1	1	1	1	1	1	1	EQU 4
EQU 5	336									10	10	10	10	10	10	10	10	EQU 5
EQU 6	1048									31	31	31	31	31	31	31	31	EQU 6
EQU 7	360									11	11	11	11	11	11	11	11	EQU 7
EQU 8	2112	10			10		10	10		51	60	60	51	60	51	51	60	EQU 8
EQU 9	37									1	1	1	1	1	1	1	1	EQU 9
EQU 10	0									1	1	1	1	1	1	1	1	EQU 10
EQU 11	6									1	1	1	1	1	1	1	1	EQU 11
EQU 12	12									1	1	1	1	1	1	1	1	EQU 12
EQU 13	24									2	2	2	2	2	2	2	2	EQU 13
EQU 14	6									1	1	1	1	1	1	1	1	EQU 14
EQU 15	29									1	1	1	1	1	1	1	1	EQU 15
EQU 16	24									4	4	4	4	4	4	4	4	EQU 16
EQU 17	1			-1	-1			-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	EQU 17
EQU 18	2			-2	-2			-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	EQU 18
EQU 19	30			2	2			2	2	2	2	2	2	2	2	2	2	EQU 19
EQU 20	1			-1	-1			-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	EQU 20
EQU 21	0			1	1			1	1	1	1	1	1	1	1	1	1	EQU 21
EQU 22	30			2	2			2	2	2	2	2	2	2	2	2	2	EQU 22
EQU 23	1318			90	90			90	90	26	26	116	116	26	26	116	116	EQU 23
EQU 24	196					8	10	10	8	-6	-6	-6	-6	2	2	2	2	EQU 24
EQU 25	33		1		1		1		1		1		1		1		1	EQU 25
EQU 26	76		1		1		1		1		1		1		1		1	EQU 26
EQU 27	7			1	1			1	1	1	1	1	1	1	1	1	1	EQU 27
EQU 28	16																	EQU 28
EQU 29	117																	EQU 29
EQU 30	111																	EQU 30
EQU 31	114																	EQU 31
EQU 32	31																	EQU 32
EQU 33	7																	EQU 33
EQU 34	1																	EQU 34
EQU 35	32																	EQU 35
EQU 36	8																	EQU 36
EQU 37	3																	EQU 37
EQU 38	12																	EQU 38
EQU 39	35																	EQU 39
EQU 40	7																	EQU 40
EQU 41	8																	EQU 41
EQU 42	87	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	EQU 42
EQU 43	7																	EQU 43
EQU 44	504									32	32	32	32	32	32	32	32	EQU 44
EQU 45	8																	EQU 45
EQU 46	240																	EQU 46
EQU 47	0																	EQU 47
CAPITAL COST		4	2	64	68	1	7	68	67	370	374	392	391	373	372	389	394	CAPITAL COST
GRAND TOTAL		139	137	230	236	136	142	235	233	560	564	581	580	563	561	578	583	GRAND TOTAL

TABLE 3-2
EQUIPMENT REQUIRED FOR SIZE 2 EXPANSION

SINGLE LINES ADDED		350	700	350	700	350	700	350	700	350	700	350	700	350	700	350	700	SINGLE LINES ADDED
DID TRUNKS ADDED		90	90	180	180	90	90	180	180	90	90	180	180	90	90	180	180	DID TRUNKS ADDED
INTRA USAGE ADDED		300	300	300	300	600	600	600	600	300	300	300	300	600	600	600	600	INTRA USAGE ADDED
INTER LOCAL ADDED		1800	1800	1800	1800	1800	1800	1800	1800	3600	3600	3600	3600	3600	3600	3600	3600	INTER LOCAL ADDED
INTER EXCHG ADDED		800	400	400	800	400	800	800	400	400	800	800	400	800	400	400	800	INTER EXCHG ADDED
DESCRIPTION	PRESENT	e	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 1
EQU 2	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	EQU 2
EQU 3	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 3
EQU 4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 4
EQU 5	336	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	EQU 5
EQU 6	1048	31	31	31	31	31	31	31	31	63	63	63	63	63	63	63	63	EQU 6
EQU 7	360	11	11	11	11	11	11	11	11	22	22	22	22	22	22	22	22	EQU 7
EQU 8	2112	70	60	60	70	60	70	70	60	111	121	121	111	121	111	111	121	EQU 8
EQU 9	37	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	EQU 9
EQU 10	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 10
EQU 11	6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 11
EQU 12	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 12
EQU 13	24	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 13
EQU 14	6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 14
EQU 15	28	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	EQU 15
EQU 16	24	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	EQU 16
EQU 17	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	EQU 17
EQU 18	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	EQU 18
EQU 19	30	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 19
EQU 20	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	EQU 20
EQU 21	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 21
EQU 22	30	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 22
EQU 23	1318	116	116	206	206	116	116	206	206	143	143	233	233	143	143	233	233	EQU 23
EQU 24	196	4	2	2	4	10	12	12	10	6	6	6	6	14	14	14	14	EQU 24
EQU 25	33	-7	-6	-7	-6	-7	-6	-7	-6	-7	-6	-7	-6	-7	-6	-7	-6	EQU 25
EQU 26	76	9	10	9	10	9	10	9	10	9	10	9	10	9	10	9	10	EQU 26
EQU 27	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 27
EQU 28	16			2	2			2	2			2	2			2	2	EQU 28
EQU 29	127		3		3		3		3		3		3		3		3	EQU 29
EQU 30	121		3		3		3		3		3		3		3		3	EQU 30
EQU 31	124		1		1		1		1		1		1		1		1	EQU 31
EQU 32	31	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	EQU 32
EQU 33	7																	EQU 33
EQU 34	1																	EQU 34
EQU 35	32																	EQU 35
EQU 36	8																	EQU 36
EQU 37	3																	EQU 37
EQU 38	12																	EQU 38
EQU 39	35		1		1		1		1		1		1		1		1	EQU 39
EQU 40	7																	EQU 40
EQU 41	8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	EQU 41
EQU 42	67	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	EQU 42
EQU 43	7																	EQU 43
EQU 44	504	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	EQU 44
EQU 45	6																	EQU 45
EQU 46	240																	EQU 46
EQU 47	0																	EQU 47
CAPITAL COST		401	407	418	431	398	411	423	428	439	451	463	469	443	449	460	473	CAPITAL COST
GRAND TOTAL		590	596	608	620	588	601	612	618	628	641	652	658	632	638	650	662	GRAND TOTAL

TABLE 3-3

EQUIPMENT REQUIRED FOR SIZE 3 EXPANSION

SINGLE LINES ADDED		700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	SINGLE LINES ADDED
DID TRUNKS ADDED		180	180	270	270	180	180	270	270	180	180	270	270	180	180	270	270	DID TRUNKS ADDED
INTRA USAGE ADDED		600	600	600	600	900	900	900	900	600	600	600	600	900	900	900	900	INTRA USAGE ADDED
INTER LOCAL ADDED		3600	3600	3600	3600	3600	3600	3600	3600	5400	5400	5400	5400	5400	5400	5400	5400	INTER LOCAL ADDED
INTER EXCHG ADDED		1200	800	800	1200	800	1200	1200	800	800	1200	1200	800	1200	800	800	1200	INTER EXCHG ADDED
DESCRIPTION	PRESENT	e	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 1
EQU 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 2
EQU 3	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 3
EQU 4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 4
EQU 5	336	20	20	20	20	20	20	20	20	30	30	30	30	30	30	30	30	EQU 5
EQU 6	1048	63	63	63	63	63	63	63	63	94	94	94	94	94	94	94	94	EQU 6
EQU 7	360	22	22	22	22	22	22	22	22	32	32	32	32	32	32	32	32	EQU 7
EQU 8	2112	130	121	121	130	121	130	130	121	171	181	181	171	181	171	171	181	EQU 8
EQU 9	37	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 9
EQU 10	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 10
EQU 11	6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 11
EQU 12	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 12
EQU 13	24	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 13
EQU 14	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 14
EQU 15	29	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 15
EQU 16	24	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	EQU 16
EQU 17	1	-1	-1			-1	-1											EQU 17
EQU 18	2	-2	-2			-2	-2											EQU 18
EQU 19	30	2	2	4	4	2	2	4	4	4	4	4	4	4	4	4	4	EQU 19
EQU 20	1	-1	-1			-1	-1											EQU 20
EQU 21	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 21
EQU 22	30	2	2	4	4	2	2	4	4	4	4	4	4	4	4	4	4	EQU 22
EQU 23	1318	233	233	323	323	233	233	323	323	259	259	349	349	259	259	349	349	EQU 23
EQU 24	196	14	14	14	14	22	22	24	22	18	18	18	18	26	26	26	26	EQU 24
EQU 25	33	-12	-10	-12	-10	-12	-10	-12	-10	-12	-10	-12	-10	-12	-10	-12	-10	EQU 25
EQU 26	76	15	16	15	16	15	16	15	16	15	16	15	16	15	16	15	16	EQU 26
EQU 27	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 27
EQU 28	16	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 28
EQU 29	117	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	EQU 29
EQU 30	111	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	EQU 30
EQU 31	114	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 31
EQU 32	31																	EQU 32
EQU 33	7																	EQU 33
EQU 34	1			1	1			1	1	1	1	1	1	1	1	1	1	EQU 34
EQU 35	32			4	4			4	4	4	4	4	4	4	4	4	4	EQU 35
EQU 36	8			1	1			1	1	1	1	1	1	1	1	1	1	EQU 36
EQU 37	3			1	1			1	1	1	1	1	1	1	1	1	1	EQU 37
EQU 38	12		1		1		1		1		1		1		1		1	EQU 38
EQU 39	35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 39
EQU 40	7		1		1		1		1		1		1		1		1	EQU 40
EQU 41	8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	EQU 41
EQU 42	67	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	EQU 42
EQU 43	7																	EQU 43
EQU 44	504	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	EQU 44
EQU 45	6																	EQU 45
EQU 46	240																	EQU 46
EQU 47	0																	EQU 47
CAPITAL COST		481	480	653	658	479	484	657	656	670	675	692	691	674	673	690	695	CAPITAL COST
GRAND TOTAL		671	670	840	845	669	674	844	843	856	862	878	877	861	859	875	882	GRAND TOTAL

TABLE 3-4

EQUIPMENT REQUIRED FOR SIZE 4 EXPANSION

SINGLE LINES ADDED		1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	SINGLE LINES ADDED
DID TRUNKS ADDED		270	270	360	360	270	270	360	360	270	270	360	360	270	270	360	360	DID TRUNKS ADDED
INTRA USAGE ADDED		900	900	900	900	1200	1200	1200	1200	900	900	900	900	1200	1200	1200	1200	INTRA USAGE ADDED
INTER LOCAL ADDED		5400	5400	5400	5400	5400	5400	5400	5400	7200	7200	7200	7200	7200	7200	7200	7200	INTER LOCAL ADDED
INTER EXCHG ADDED		1600	1200	1200	1600	1200	1600	1600	1200	1200	1600	1600	1200	1600	1200	1200	1600	INTER EXCHG ADDED
DESCRIPTION	PRESENT	e	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 1
EQU 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 2
EQU 3	8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 3
EQU 4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 4
EQU 5	336	30	30	30	30	30	30	30	30	40	40	40	40	40	40	40	40	EQU 5
EQU 6	1048	94	94	94	94	94	94	94	94	126	126	126	126	126	126	126	126	EQU 6
EQU 7	360	32	32	32	32	32	32	32	32	43	43	43	43	43	43	43	43	EQU 7
EQU 8	2112	191	181	181	191	181	191	191	181	232	242	242	232	242	232	232	242	EQU 8
EQU 9	37	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	EQU 9
EQU 10	0	1	1	1	1	1	1	1	1									EQU 10
EQU 11	6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 11
EQU 12	12	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	EQU 12
EQU 13	24	2	2	2	2	2	2	2	2	4	4	4	4	4	4	4	4	EQU 13
EQU 14	6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 14
EQU 15	29	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	EQU 15
EQU 16	24	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	EQU 16
EQU 17	1																	EQU 17
EQU 18	2																	EQU 18
EQU 19	30	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	EQU 19
EQU 20	1																	EQU 20
EQU 21	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 21
EQU 22	30	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	EQU 22
EQU 23	1318	349	349	439	439	349	349	439	439	375	375	465	465	375	375	465	465	EQU 23
EQU 24	196	26	26	26	26	34	36	36	34	-16	-16	-16	-16	-8	-8	-8	-8	EQU 24
EQU 25	33	-16	-15	-16	-15	-16	-15	-16	-15	-16	-15	-16	-15	-16	-15	-16	-15	EQU 25
EQU 26	76	21	22	21	22	21	22	21	22	21	22	21	22	21	22	21	22	EQU 26
EQU 27	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 27
EQU 28	16	2	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4	EQU 28
EQU 29	117	3	6	3	6	3	6	3	6	3	6	3	6	3	6	3	6	EQU 29
EQU 30	121	3	6	3	6	3	6	3	6	3	6	3	6	3	6	3	6	EQU 30
EQU 31	114	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 31
EQU 32	31	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	EQU 32
EQU 33	7			1	1			1	1			1	1			1	1	EQU 33
EQU 34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 34
EQU 35	32	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	EQU 35
EQU 36	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 36
EQU 37	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 37
EQU 38	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 38
EQU 39	35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 39
EQU 40	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	EQU 40
EQU 41	8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	EQU 41
EQU 42	67	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	EQU 42
EQU 43	7																	EQU 43
EQU 44	504	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	EQU 44
EQU 45	6									1	1	1	1	1	1	1	1	EQU 45
EQU 46	240									96	96	96	96	96	96	96	96	EQU 46
EQU 47	8									1	1	1	1	1	1	1	1	EQU 47
CAPITAL COST		702	700	808	812	700	704	813	810	944	948	1056	1054	947	945	1053	1057	CAPITAL COST
GRAND TOTAL		889	887	995	999	887	891	1000	997	1154	1158	1266	1264	1157	1155	1263	1267	GRAND TOTAL

TABLE 3-5

DIFFERENTIAL EQUIPMENT REQUIRED FOR SIZE 2 EXPANSION

SINGLE LINES ADDED		350	700	350	700	350	700	350	700	350	700	350	700	350	700	350	700	SINGLE LINES ADDED
DID TRUNKS ADDED		90	90	180	180	90	90	180	180	90	90	180	180	90	90	180	180	DID TRUNKS ADDED
INTRA USAGE ADDED		300	300	300	300	600	600	600	600	300	300	300	300	600	600	600	600	INTRA USAGE ADDED
INTER LOCAL ADDED		1800	1800	1800	1800	1800	1800	1800	1800	3600	3600	3600	3600	3600	3600	3600	3600	INTER LOCAL ADDED
INTER EXCHG ADDED		800	400	400	800	400	800	800	400	400	800	800	400	800	400	400	800	INTER EXCHG ADDED
DESCRIPTION	PRESENT	e	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3																	EQU 1
EQU 2	2																	EQU 2
EQU 3	0																	EQU 3
EQU 4	1																	EQU 4
EQU 5	336									10	10	10	10	10	10	10	10	EQU 5
EQU 6	1048									32	32	32	32	32	32	32	32	EQU 6
EQU 7	360									11	11	11	11	11	11	11	11	EQU 7
EQU 8	2112	10			10		10	10		51	61	61	51	61	51	51	61	EQU 8
EQU 9	37									1	1	1	1	1	1	1	1	EQU 9
EQU 10	0																	EQU 10
EQU 11	6																	EQU 11
EQU 12	12																	EQU 12
EQU 13	24																	EQU 13
EQU 14	6																	EQU 14
EQU 15	29									1	1	1	1	1	1	1	1	EQU 15
EQU 16	24																	EQU 16
EQU 17	1																	EQU 17
EQU 18	2																	EQU 18
EQU 19	30																	EQU 19
EQU 20	1																	EQU 20
EQU 21	0																	EQU 21
EQU 22	30																	EQU 22
EQU 23	1318			90	90			90	90	27	27	117	117	27	27	117	117	EQU 23
EQU 24	196	2			2	8	10	10	8	4	4	4	4	12	12	12	12	EQU 24
EQU 25	33	-8	-7	-8	-7	-8	-7	-8	-7	-8	-7	-8	-7	-8	-7	-8	-7	EQU 25
EQU 26	76	8	9	8	9	8	9	8	9	8	9	8	9	8	9	8	9	EQU 26
EQU 27	7																	EQU 27
EQU 28	16			2	2			2	2			2	2			2	2	EQU 28
EQU 29	117		3		3		3		3		3		3		3		3	EQU 29
EQU 30	111		3		3		3		3		3		3		3		3	EQU 30
EQU 31	114		1		1		1		1		1		1		1		1	EQU 31
EQU 32	31	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	EQU 32
EQU 33	7																	EQU 33
EQU 34	1																	EQU 34
EQU 35	32																	EQU 35
EQU 36	8																	EQU 36
EQU 37	3																	EQU 37
EQU 38	12																	EQU 38
EQU 39	35		1		1		1		1		1		1		1		1	EQU 39
EQU 40	7																	EQU 40
EQU 41	8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	-8	EQU 41
EQU 42	87																	EQU 42
EQU 43	7																	EQU 43
EQU 44	504																	EQU 44
EQU 45	8																	EQU 45
EQU 46	240																	EQU 46
EQU 47	0																	EQU 47
CAPITAL COST		7	13	24	37	4	17	29	34	45	57	68	73	49	55	66	78	CAPITAL COST
GRAND TOTAL		7	13	25	37	5	18	29	35	45	58	68	75	49	55	67	79	GRAND TOTAL

TABLE 3-6

DIFFERENTIAL EQUIPMENT REQUIRED FOR SIZE 3 EXPANSION

SINGLE LINES ADDED		700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	SINGLE LINES ADDED
DID TRUNKS ADDED		180	180	270	270	180	180	270	270	180	180	270	270	180	180	270	270	DID TRUNKS ADDED
INTRA USAGE ADDED		600	600	600	600	900	900	900	900	600	600	600	600	900	900	900	900	INTRA USAGE ADDED
INTER LOCAL ADDED		3600	3600	3600	3600	3600	3600	3600	3600	5400	5400	5400	5400	5400	5400	5400	5400	INTER LOCAL ADDED
INTER EXCHG ADDED		1200	800	800	1200	800	1200	1200	800	800	1200	1200	800	1200	800	800	1200	INTER EXCHG ADDED
DESCRIPTION	PRESENT	o	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3																	EQU 1
EQU 2	2																	EQU 2
EQU 3	0																	EQU 3
EQU 4	1																	EQU 4
EQU 5	336									10	10	10	10	10	10	10	10	EQU 5
EQU 6	1048									31	31	31	31	31	31	31	31	EQU 6
EQU 7	360									10	10	10	10	10	10	10	10	EQU 7
EQU 8	2112	9			9		9	9		50	60	60	50	60	50	50	60	EQU 8
EQU 9	37																	EQU 9
EQU 10	0																	EQU 10
EQU 11	6																	EQU 11
EQU 12	12																	EQU 12
EQU 13	24																	EQU 13
EQU 14	6																	EQU 14
EQU 15	29																	EQU 15
EQU 16	24																	EQU 16
EQU 17	1			1	1			1	1	1	1	1	1	1	1	1	1	EQU 17
EQU 18	2			2	2			2	2	2	2	2	2	2	2	2	2	EQU 18
EQU 19	30			2	2			2	2	2	2	2	2	2	2	2	2	EQU 19
EQU 20	1			1	1			1	1	1	1	1	1	1	1	1	1	EQU 20
EQU 21	0																	EQU 21
EQU 22	30			2	2			2	2	2	2	2	2	2	2	2	2	EQU 22
EQU 23	1318			90	90			90	90	26	26	116	116	26	26	116	116	EQU 23
EQU 24	196					8	8	10	8	4	4	4	4	12	12	12	12	EQU 24
EQU 25	33	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	EQU 25
EQU 26	78	5	6	5	6	5	6	5	6	5	6	5	6	5	6	5	6	EQU 26
EQU 27	7																	EQU 27
EQU 28	16																	EQU 28
EQU 29	117																	EQU 29
EQU 30	111																	EQU 30
EQU 31	114																	EQU 31
EQU 32	31	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EQU 32
EQU 33	7																	EQU 33
EQU 34	1			1	1			1	1	1	1	1	1	1	1	1	1	EQU 34
EQU 35	32			4	4			4	4	4	4	4	4	4	4	4	4	EQU 35
EQU 36	8			1	1			1	1	1	1	1	1	1	1	1	1	EQU 36
EQU 37	3			1	1			1	1	1	1	1	1	1	1	1	1	EQU 37
EQU 38	12		1		1		1		1		1		1		1		1	EQU 38
EQU 39	35																	EQU 39
EQU 40	7		1		1		1		1		1		1		1		1	EQU 40
EQU 41	8																	EQU 41
EQU 42	67																	EQU 42
EQU 43	7																	EQU 43
EQU 44	504																	EQU 44
EQU 45	6																	EQU 45
EQU 46	240																	EQU 46
EQU 47	0																	EQU 47
CAPITAL COST		8	7	180	185	6	11	184	183	197	202	219	218	201	200	217	222	CAPITAL COST
GRAND TOTAL		9	8	176	183	7	12	182	181	194	200	216	215	199	197	213	220	GRAND TOTAL

TABLE 3-7

DIFFERENTIAL EQUIPMENT REQUIRED FOR SIZE 4 EXPANSION

SINGLE LINES ADDED		1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	1050	1400	SINGLE LINES ADDED
DID TRUNKS ADDED		270	270	360	360	270	270	360	360	270	270	360	360	270	270	360	360	DID TRUNKS ADDED
INTRA USAGE ADDED		900	900	900	900	1200	1200	1200	1200	900	900	900	900	1200	1200	1200	1200	INTRA USAGE ADDED
INTER LOCAL ADDED		5400	5400	5400	5400	5400	5400	5400	5400	7200	7200	7200	7200	7200	7200	7200	7200	INTER LOCAL ADDED
INTER EXCHG ADDED		1600	1200	1200	1600	1200	1600	1600	1200	1200	1600	1600	1200	1600	1200	1200	1600	INTER EXCHG ADDED
DESCRIPTION	PRESENT	e	a	b	abe	c	ace	bce	abc	d	ade	bde	abd	cde	acd	bcd	abcde	DESCRIPTION
EQU 1	3																	EQU 1
EQU 2	2																	EQU 2
EQU 3	0																	EQU 3
EQU 4	1																	EQU 4
EQU 5	336									10	10	10	10	10	10	10	10	EQU 5
EQU 6	1048									32	32	32	32	32	32	32	32	EQU 6
EQU 7	360									11	11	11	11	11	11	11	11	EQU 7
EQU 8	2112	10			10		10	10		51	61	61	51	61	51	51	61	EQU 8
EQU 9	37									1	1	1	1	1	1	1	1	EQU 9
EQU 10	0									-1	-1	-1	-1	-1	-1	-1	-1	EQU 10
EQU 11	6																	EQU 11
EQU 12	12									1	1	1	1	1	1	1	1	EQU 12
EQU 13	24									2	2	2	2	2	2	2	2	EQU 13
EQU 14	6																	EQU 14
EQU 15	29									1	1	1	1	1	1	1	1	EQU 15
EQU 16	24																	EQU 16
EQU 17	1																	EQU 17
EQU 18	2																	EQU 18
EQU 19	30																	EQU 19
EQU 20	1																	EQU 20
EQU 21	0																	EQU 21
EQU 22	30																	EQU 22
EQU 23	1318			90	90			90	90	26	26	116	116	26	26	116	116	EQU 23
EQU 24	196					8	10	10	8	-42	-42	-42	-42	-34	-34	-34	-34	EQU 24
EQU 25	33	-6	-5	-6	-5	-6	-5	-6	-5	-6	-5	-6	-5	-6	-5	-6	-5	EQU 25
EQU 26	76	5	6	5	6	5	6	5	6	5	6	5	6	5	6	5	6	EQU 26
EQU 27	7																	EQU 27
EQU 28	16			2	2			2	2			2	2			2	2	EQU 28
EQU 29	117		3		3		3		3		3		3		3		3	EQU 29
EQU 30	111		3		3		3		3		3		3		3		3	EQU 30
EQU 31	114																	EQU 31
EQU 32	31	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	EQU 32
EQU 33	7			1	1			1	1			1	1			1	1	EQU 33
EQU 34	1																	EQU 34
EQU 35	32																	EQU 35
EQU 36	8																	EQU 36
EQU 37	3																	EQU 37
EQU 38	12																	EQU 38
EQU 39	35																	EQU 39
EQU 40	7																	EQU 40
EQU 41	8																	EQU 41
EQU 42	67																	EQU 42
EQU 43	7																	EQU 43
EQU 44	504																	EQU 44
EQU 45	6									1	1	1	1	1	1	1	1	EQU 45
EQU 46	240									96	96	96	96	96	96	96	96	EQU 46
EQU 47	0									1	1	1	1	1	1	1	1	EQU 47
CAPITAL COST		7	5	113	117	5	9	118	115	248	253	361	358	252	250	354	362	CAPITAL COST
GRAND TOTAL		7	5	113	117	5	9	118	115	272	276	384	382	275	273	381	385	GRAND TOTAL

TABLE 3-8

EXAMPLE OF FLAWED DIFFERENTIAL SIZE 3 EXPANSION

SINGLE LINES ADDED		700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	700	1050	SINGLE LINES ADDED
DID TRUNKS ADDED		180	180	270	270	180	180	270	270	180	180	270	270	180	180	270	270	DID TRUNKS ADDED
INTRA USAGE ADDED		600	600	600	600	900	900	900	900	600	600	600	600	900	900	900	900	INTRA USAGE ADDED
INTER LOCAL ADDED		3600	3600	3600	3600	3600	3600	3600	3600	5400	5400	5400	5400	5400	5400	5400	5400	INTER LOCAL ADDED
INTER EXCHG ADDED		1200	800	800	1200	800	1200	1200	800	800	1200	1200	800	1200	800	800	1200	INTER EXCHG ADDED
DESCRIPTION	PRESENT	o	a	b	ab	c	ac	bc	abc	d	ad	bd	abd	cd	acd	bcd	abcd	DESCRIPTION
EDU 1	3																	EDU 1
EDU 2	2																	EDU 2
EDU 3	0																	EDU 3
EDU 4	1																	EDU 4
EDU 5	336									10	10	10	10	10	10	10	10	EDU 5
EDU 6	1048									31	31	31	31	31	31	31	31	EDU 6
EDU 7	360									10	10	10	10	10	10	10	10	EDU 7
EDU 8	2112	0			0		0	0		50	60	60	20 (circled)	30 (circled)	50	50	60	EDU 8
EDU 9	37																	EDU 9
EDU 10	0																	EDU 10
EDU 11	6																	EDU 11
EDU 12	12																	EDU 12
EDU 13	24																	EDU 13
EDU 14	6																	EDU 14
EDU 15	29																	EDU 15
EDU 16	24																	EDU 16
EDU 17	1			1				1	1	1	1	1	1	1	1	1	1	EDU 17
EDU 18	2			2	2			2	2	2	2	2	2	2	2	2	2	EDU 18
EDU 19	30			2	2			2	2	2	2	2	2	2	2	2	2	EDU 19
EDU 20	1			1	1			1	1	1	1	1	1	1	1	1	1	EDU 20
EDU 21	0																	EDU 21
EDU 22	38			2	2			2	2	2	2	2	2	2	2	2	2	EDU 22
EDU 23	1318			90	90			90	90	26	26	116	116	26	26	116	116	EDU 23
EDU 24	196					8	8	10	8	4	4	4	4	12	12	12	12	EDU 24
EDU 25	33	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	-6	-4	EDU 25
EDU 26	76	5	6	5	6	5	6	5	6	5	6	5	6	5	6	5	6	EDU 26
EDU 27	7																	EDU 27
EDU 28	16																	EDU 28
EDU 29	127																	EDU 29
EDU 30	121																	EDU 30
EDU 31	114																	EDU 31
EDU 32	21	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	EDU 32
EDU 33	7																	EDU 33
EDU 34	1			1	1			1	1	1	1	1	1	1	1	1	1	EDU 34
EDU 35	32			4	4			4	4	4	4	4	4	4	4	4	4	EDU 35
EDU 36	8			1	1			1	1	1	1	1	1	1	1	1	1	EDU 36
EDU 37	3			1	1			1	1	1	1	1	1	1	1	1	1	EDU 37
EDU 38	12		1		1		1		1		1		1		1		1	EDU 38
EDU 39	35																	EDU 39
EDU 40	7		1		1		1		1		1		1		1		1	EDU 40
EDU 41	8																	EDU 41
EDU 42	67																	EDU 42
EDU 43	7																	EDU 43
EDU 44	504																	EDU 44
EDU 45	6																	EDU 45
EDU 46	240																	EDU 46
EDU 47	0																	EDU 47
CAPITAL COST		207	208	233	238	207	212	237	236	250	255	272	271	254	253	270	275	CAPITAL COST
GRAND TOTAL		210	209	231	236	208	213	235	234	247	253	269	268	252	250	266	273	GRAND TOTAL

Step 5: Analyze

Regardless which experimental plan is used, four sets of data are obtained, as shown in tables 3-9 through 3-12 for the pilot study. Each set follows a fractional factorial format for each of the four sizes of expansions. One analysis approach is to combine all four sets of data into a composite set and fit a general linear model to the data. When one examines the costs over the entire range represented by all four sizes, considerable curvature is found. In the pilot study attempts were made to account for the curvature by using quadratic models and by using logarithm transformations. The most successful of these models used quadratic terms in the five variables and no linear terms. It used an indicator variable to account for the addition of the LLN in the size 1, 2, and 3 expansions, and another indicator variable to account for the addition of the larger LLN in the size 4 expansions.

A much more successful approach to treat the curvature in the data was to fit a piecewise model (also called spline function) to the data with each piece being fit to the fractional factorial data that is obtained for each size expansion. The spline functions that best fit the pilot study data contained four pieces, each consisting of linear terms for each variable. In the size 1 and size 3 pieces a two variable cross-product term was also needed. Standard least squares procedures are modified for use in fitting spline functions to estimate the parameters of the model. The reason spline function models were the preferred models is that they can fit well a rather complex surface with several functional pieces with each having a very simple mathematical structure. This makes them easy to use. It was also the case in the pilot that they resulted in extremely small mean square errors.

Recall that the expansion plan for a size 1 expansion denoted "abcde" is exactly the same expansion plan that is denoted "(1)" in the size 2 expansions. The same occurs at the boundary point between the size 2 and size 3 expansions and between the size 3 and size 4 expansions. When spline functions are used these common points are called knot points. Each knot point results in a constraint being added to the ordinary least squares problem that is typically used to fit models to data. Basically, these

TABLE 3-9

DATA SET FOR SIZE 1 EXPANSION

SINGLE LINES ADDED	DID TRUNKS ADDED	INTRAOFFICE ADDED	INTEROFFICE LOCAL ADDED	INTEROFFICE TOLL ADDED	CAPITAL COST (THOUSANDS)	TOTAL COST (THOUSANDS)
0	0	0	0	400	4	139
350	0	0	0	0	2	137
0	90	0	0	0	64	230
350	90	0	0	400	69	236
0	0	300	0	0	1	136
350	0	300	0	400	7	142
0	90	300	0	400	69	235
350	90	300	0	0	67	233
0	0	0	1800	0	370	560
350	0	0	1800	400	374	564
0	90	0	1800	400	392	581
350	90	0	1800	0	391	580
0	0	300	1800	400	373	563
350	0	300	1800	0	372	561
0	90	300	1800	0	389	578
350	90	300	1800	400	394	583

TABLE 3-10

DATA SET FOR SIZE 2 EXPANSION

SINGLE LINES ADDED	DID TRUNKS ADDED	INTRAOFFICE ADDED	INTEROFFICE LOCAL ADDED	INTEROFFICE TOLL ADDED	CAPITAL COST (THOUSANDS)	TOTAL COST (THOUSANDS)
350	90	300	1800	800	401	590
700	90	300	1800	400	407	596
350	180	300	1800	400	418	608
700	180	300	1800	800	431	620
350	90	600	1800	400	398	588
700	90	600	1800	800	411	601
350	180	600	1800	800	423	612
700	180	600	1800	400	428	618
350	90	300	3600	400	439	628
700	90	300	3600	800	451	641
350	180	300	3600	800	463	652
700	180	300	3600	400	469	658
350	90	600	3600	800	443	632
700	90	600	3600	400	449	638
350	180	600	3600	400	460	650
700	180	600	3600	800	473	662

TABLE 3-11

DATA SET FOR SIZE 3 EXPANSION

SINGLE LINES ADDED	DID TRUNKS ADDED	INTRAOFFICE ADDED	INTEROFFICE LOCAL ADDED	INTEROFFICE TOLL ADDED	CAPITAL COST (THOUSANDS)	TOTAL COST (THOUSANDS)
700	180	600	3600	1200	481	671
1050	180	600	3600	800	480	670
700	270	600	3600	800	653	840
1050	270	600	3600	1200	658	845
700	180	900	3600	800	479	669
1050	180	900	3600	1200	484	674
700	270	900	3600	1200	657	844
1050	270	900	3600	800	656	843
700	180	600	5400	800	670	856
1050	180	600	5400	1200	675	862
700	270	600	5400	1200	692	878
1050	270	600	5400	800	691	877
700	180	900	5400	1200	674	861
1050	180	900	5400	800	673	859
700	270	900	5400	800	690	875
1050	270	900	5400	1200	695	882

TABLE 3-12

DATA SET FOR SIZE 4 EXPANSION

SINGLE LINES ADDED	DID TRUNKS ADDED	INTRAOFFICE ADDED	INTEROFFICE LOCAL ADDED	INTEROFFICE TOLL ADDED	CAPITAL COST (THOUSANDS)	TOTAL COST (THOUSANDS)
1050	270	900	5400	1600	702	889
1400	270	900	5400	1200	700	887
1050	360	900	5400	1200	808	995
1400	360	900	5400	1600	812	999
1050	270	1200	5400	1200	700	887
1400	270	1200	5400	1600	704	891
1050	360	1200	5400	1600	813	1000
1400	360	1200	5400	1200	810	997
1050	270	900	7200	1200	944	1154
1400	270	900	7200	1600	948	1158
1050	360	900	7200	1600	1056	1266
1400	360	900	7200	1200	1054	1264
1050	270	1200	7200	1600	947	1157
1400	270	1200	7200	1200	945	1155
1050	360	1200	7200	1200	1053	1263
1400	360	1200	7200	1600	1057	1267

constraints force all adjacent size pieces of the spline model to agree at their common boundary points.

An alternative to using the constrained least squares approach is to fit the models independently and ignore any model disagreements at their boundaries. In fact these disagreements were very small. In this approach one would use the model for the smaller size expansions to evaluate a point that is common to two adjacent sizes. Our preference is to add restriction to the procedures that fit the models rather than add restrictions to how the models may be used.

The pilot data presented in tables 3-9 to 3-12 were analyzed using a constrained least squares technique closely related to (but not the same as) that defined for spline functions. The procedure was developed to use existing software to approximate the spline function calculations. It has subsequently been found that some versions of the SAS statistical analysis system contain procedures that will allow direct computation of the spline functions. When the direct calculation method has been applied to the pilot data, the results differed from those presented here by insignificant amounts.

The models derived from the pilot data using the constrained least squares approach are given in tables 3-13 and 3-14. In these models the following variable definitions are used:

x_1 is the number of access lines added,

x_2 is the number of interswitch local busy-hour CCS added,

x_3 is the number of intraswitch busy-hour CCS added,

x_4 is the number of DID trunks added,

x_5 is the number of interexchange busy-hour CCS added,

y_c is the capital cost (defined for income tax purposes) of an expansion, and

y_t is the total cost of an expansion.

The figures 3-2 through 3-9 were obtained by summing across the four pieces of each model only those terms involving the variable being graphed. For

TABLE 3-13

THE COEFFICIENTS OF THE CAPITAL COST (y_c) MODEL OF SWITCH EXPANSIONS

Intercept	x_1	x_2	x_3	x_4	x_5	x_2x_4	Variable Ranges
Size 1: 0	5.51	205.10	3.10	712.30	8.75	-.277	$0 < x_1 < 350$
							$0 < x_2 < 1800$
							$0 < x_3 < 300$
							$0 < x_4 < 90$
							$0 < x_5 < 400$
Size 2: 313,436	28.76	23.83	5.22	248.70	10.79	0	$351 < x_1 < 700$
							$1801 < x_2 < 3600$
							$301 < x_3 < 600$
							$91 < x_4 < 180$
							$401 < x_5 < 800$
Size 3: -908,268	7.54	282.12	5.46	5453.51	9.10	-.971	$701 < x_1 < 1050$
							$3601 < x_2 < 5400$
							$601 < x_3 < 900$
							$181 < x_4 < 270$
							$801 < x_5 < 1200$
Size 3: -398,564	5.14	136.86	5.16	1234.41	10.12	0	$1051 < x_1 < 1400$
							$5401 < x_2 < 7200$
							$901 < x_3 < 1200$
							$271 < x_4 < 360$
							$1201 < x_5 < 1600$

Source: Authors' calculation using Ohio Bell data

TABLE 3-14

THE COEFFICIENTS OF THE TOTAL COST (y_t) MODEL OF SWITCH EXPANSIONS

Intercept	x_1	x_2	x_3	x_4	x_5	x_2x_4	Variable Ranges
Size 1: 135,625	5.00	235.125	1.67	1055.55	8.75	-.472	$0 < x_1 < 350$
							$0 < x_2 < 1800$
							$0 < x_3 < 300$
							$0 < x_4 < 90$
							$0 < x_5 < 400$
Size 2: 500,891	29.38	23.93	6.78	253.52	10.70	0	$351 < x_1 < 700$
							$1801 < x_2 < 3600$
							$301 < x_3 < 600$
							$91 < x_4 < 180$
							$401 < x_5 < 800$
Size 3: -708,638	9.54	278.91	6.96	5394.88	11.47	-.961	$701 < x_1 < 1050$
							$3601 < x_2 < 5400$
							$601 < x_3 < 900$
							$181 < x_4 < 270$
							$801 < x_5 < 1200$
Size 3: -280,234	5.10	149.62	5.12	1234.13	10.09	0	$1051 < x_1 < 1400$
							$5401 < x_2 < 7200$
							$901 < x_3 < 1200$
							$271 < x_4 < 360$
							$1201 < x_5 < 1600$

Source: Authors' calculation using Ohio Bell data

TABLE 3-15

THREE CASES FOR THE VALUE OF x_4
WHILE COMPUTING x_2'S COST

Case	Size 1	Size 2	Size 3	Size 4
1	0	90	180	270
2	90	90	180	270
3	90	90	270	270

Source: Authors' assumption

example, consider access lines which was the variable of interest for figure 3-2. The contribution to cost of access lines is computed at $5.51 for each of the first 350 lines, at $28.76 for each of the next 350 lines, at $7.54 for each line over 700 lines added but less than 1051 lines, and the contribution of the last 350 lines added is computed at $5.14 each. A similar calculation is made for the interswitch local busy-hour traffic, but due to the cross product term, x_2x_4, in the size 1 and size 3 pieces of the model, its contribution to cost is calculated under three cases given in table 3-15.

Because the coefficient for the x_2x_4 term is negative in the size 1 and size 3 pieces of the models, a savings occurs when both DID trunks and capacity for interswitch local busy-hour are added to the switch. The calculations made for figures 3-4, 3-5, 3-8, and 3-9 allocated the entire savings to the interswitch traffic variable. An alternate calculation could have allocated the savings in some manner to both. An argument that favors the former method is that a business customer adds DID trunks because he needs to pass more traffic through the system, including the interswitch trunking. Thus, a cost savings resulting from a lower cost on the traffic variable benefits the business customer when the total cost of all his services is computed.

Some variables have an inconsistent effect on costs, both the capital cost and the total cost. This is primarily due to the "lumpiness" of some items of equipment as discussed earlier. One should also note that the average cost curves tend to stabilize toward the right side of the graphs. This is because as one moves to the right on the average cost curves, the average is being taken over a larger and larger number of units. This has the effect of smoothing out the lumpiness. When a sample of several switches is used, an average over the switches should have the same effect. Turn now to a process of averaging over the several switches in a sample, and to the computation of a customer cost.

Step 6: Average

The computation of an average incremental cost[4] for a customer class is relatively complicated. It requires the use of the CAPCOST model that was developed as part of this project. The CAPCOST model is described in Appendix D. The averaging technique also requires growth forecasts for each switch in the sample. We shall explain the method by working through an example in which we assume a hypothetical sample of three switches. One of the three switches will be the one used in the pilot work but its growth forecast is hypothetical. In the calculations that follow, any savings from the joint addition of DID trunks and interswitch busy-hour traffic capacity is all allocated to the interswitch busy-hour traffic capacity expansion. One switch in the sample will be losing customers while the other two will be gaining customers. For those switches gaining customers an initial step is to "optimize" the expansion size. An average cost per unit for that size

[4] Average incremental cost is the quantity that we estimate and then use as a proxy for marginal cost. In fact, increments of capacity are added in a switch expansion rather than marginal units because the equipment is not infinitely divisible, thus a true marginal-cost calculation is not possible.

expansion is then computed. For the switch that is losing customers the historical average size of expansions is used to compute the average cost per unit. Once the average cost per unit is calculated, a weighted average is then calculated across switches. The weight for a switch is (under an independence assumption) the probability that a unit of capacity change occurs at that switch during the next year. We now proceed with the example using the growth data given in table 3-16.

TABLE 3-16

FORECASTED GROWTH BY OFFICE: HYPOTHETICAL DATA

		Year Into the Future						
Switch	Variable	1	2	3	4	5	6	7
#1 (Pilot)	x_1	250	300	350	400	400	400	400
	x_2	1200	1500	2000	2400	2600	2600	2600
	x_3	200	250	300	350	350	350	350
#2	x_1	-100	-100	-100	-100	-100	-100	-100
	x_2	-600	-600	-600	-600	-600	-600	-600
	x_3	-50	-50	-50	-50	-50	-50	-50
#3	x_1	400	500	600	500	400	300	200
	x_2	2000	2400	2800	3600	4000	3500	3000
	x_3	500	550	600	650	650	600	600

Source: Authors' assumptions

Note: x_1 stands for access lines.
x_2 stands for interswitch local busy-hour CCS.
x_3 stands for intraswitch busy-hour CCS.

The optimization process will be demonstrated on switch #1 and will minimize the annual equivalent cost per line for the entire expansion. Only first year construction plans will be considered for the optimization problem. Thus, the construction plan decision consists of determining how many years of growth should be satisfied by the plant added in the first year. The total cost of each construction plan can be determined from the models given in table 3-14, or more easily by adding component costs read from the curves given in figures 3-6 through 3-9. Figures 3-2 to 3-5 can be used to obtain the corresponding capital cost, or the models in table 3-13 can be used for the same purpose. It should be noted that in figures 3-4 and 3-8 there are several curves to consider. We based the cost contribution of interswitch traffic on two situations. In the first, we assumed an annual demand for new DID trunks that was large enough to gain the economies that are possible from the joint addition of trunks and capacity for interswitch busy-hour traffic (i.e., we used the lower most curve). In the second, we assumed no such economies because of insufficient demand for DID trunks (i.e., we used the uppermost curve). The resulting values can then be weighted with either their historical or their forecasted probabilities of occurring, and added together to obtain a composite figure. For this example, the weights we used were 0.5 and 0.5.

In this step the CAPCOST model is now run for each construction plan using the capital cost as an input for the initial investment, and the difference between the total cost and the capital cost as the entry for a special expense in the first year. The fill rate will vary depending on which plan is assumed, but it will be based on the number of lines added. As a result, the one year plan uses a fill rate of 1 in each year, while the two year plan will be filled to $(250/550)^{ths}$ of its capacity the first year and will have a fill rate of 1 in all subsequent years. The three year plan will have fill rates of 250/900 the first year, 550/900 the second year, and 1 each year thereafter. At the present time, operation and maintenance (O & M) costs have not been determined in this study so that they should be set to zero for this step. It is unlikely that O & M costs will have an effect on the optimal first year construction plan, although they will affect the cost that must be recovered. Construction plans that go beyond four or five years should not be considered because of the extreme uncertainty about forecasts that far into the future. The CAPCOST solution is divided by the number of lines added in the corresponding construction plan.

Table 3-17 gives the result of a CAPCOST run on the one year expansion plan. The solution is divided by 250 to obtain an average annual per line

TABLE 3-17

CAPCOST MODEL OUTPUT

Capital Structure:		Other Information: Horizon	= 15
Percentage composed of debt	= 40.0%	Initial amount invested	= $257,160
Average interest rate on debt	= 9.0%	Regulatory book life	= 10
Percent composed of equity	= 60.0%	Tax life	= 5
Avg rate of return on equity	= 16.0%	Tax depreciation method	= ACRS
Composite Cost of Capital	= 13.2%	O&M growth rate	= 0.0%
Composite Income Tax Rate	= 46.0%	O&M gradient change	= $0.00
		O&M initial cost	= $0.00
Solution:		Gompertz-Makeham parameters:	
Lower Bound: $65,614.04	E NPV UI:	s =	0
Mid Point: $65,614.31	($0.7634)	g =	0
Upper Bound: $65,614.57	Req'd Width:	c =	0
Width: $ 0.531426	$1.000000		

YEAR A	FILL RATE B	REVENUES C	SPECIAL EXPENSES D	O & M COSTS E	BOOK DEP F	ACC BOOK DEP G	TAX DEP H	DEFERRED TAXES I	ACC TAX DEFERRALS J	YEAR K
1	1	$65,614	$150,040	$0	$25,716	$ 25,716	$51,432	$11,829	$11,829	1
2	1	65,614	0	0	25,716	51,432	82,291	26,025	37,854	2
3	1	65,614	0	0	25,716	77,148	61,718	16,561	54,415	3
4	1	65,614	0	0	25,716	102,864	41,146	7,098	61,513	4
5	1	65,614	0	0	25,716	128,580	20,573	(2,366)	59,147	5
6	1	65,614	0	0	25,716	154,296	0	(11,829)	47,317	6
7	1	65,614	0	0	25,716	180,012	0	(11,829)	35,488	7
8	1	65,614	0	0	25,716	205,728	0	(11,829)	23,659	8
9	1	65,614	0	0	25,716	231,444	0	(11,829)	11,829	9
10	1	65,614	0	0	25,716	257,160	0	(11,829)	0	10
11	1	65,614	0	0	0	257,160	0	0	0	11
12	1	65,614	0	0	0	257,160	0	0	0	12
13	1	65,614	0	0	0	257,160	0	0	0	13
14	1	65,614	0	0	0	257,160	0	0	0	14
15	1	65,614	0	0	0	257,160	0	0	0	15

TABLE 3-17 (continued)

YEAR L	RATE BASE M	INTEREST ON DEBT (ACTUAL) N	INTEREST ON DEBT (ALLOWED) O	TAXABLE INCOME P	INCOME TAX Q	AFTER TAX CASH FLOW R	EQUITY INCOME (ACTUAL) S	EQUITY INCOME (ALLOWED) T	YEAR U
1	$219,615	$ 9,258	$ 9,258	($145,115)	($66,753)	($17,673)	$24,687	$24,687	1
2	167,874	11,116	7,906	(27,793)	(12,785)	78,399	29,643	21,083	2
3	125,597	9,761	6,043	(5,865)	(2,698)	68,312	26,029	16,116	3
4	92,783	8,590	4,521	15,879	7,304	58,310	22,907	12,057	4
5	69,433	7,625	3,340	37,417	17,212	48,403	20,333	8,907	5
6	55,547	6,889	2,500	58,725	27,014	38,601	18,370	6,666	6
7	41,660	6,409	2,000	59,206	27,235	38,380	17,090	5,332	7
8	27,773	5,873	1,500	59,741	27,481	38,133	15,661	3,999	8
9	13,887	5,275	1,000	60,339	27,756	37,858	14,067	2,666	9
10	0	4,609	500	61,006	28,063	37,552	12,290	1,333	10
11	0	3,865	0	61,749	28,405	37,210	10,307	0	11
12	0	3,036	0	62,578	28,786	36,828	8,096	0	12
13	0	2,111	0	63,504	29,212	36,403	5,629	0	13
14	0	1,079	0	64,535	29,686	35,928	2,877	0	14
15	0	(72)	0	65,686	30,216	35,399	(192)	0	15

YEAR V	CAPITAL CONSUMPTION W	UNRECOVERED INVESTMENT X	REGULATORY SURPLUS Y	ACC FW OF SURPLUS Z	UNRECOVERD REGULATED INVESTMENT AA	PROBABILITY OF RETIREMENT IN THE YEAR AB	PROBABILITY UNRECOVERD (ACTUAL) AC	WEIGHTED INVESTMENT (REGULATED) AD	YEAR AE
1	($51,618)	$308,778	($ 89,163)	($ 89,163)	$308,778	0.00	$ 30.88	$ 30.88	1
2	37,640	271,137	(2,331)	(103,263)	271,137	0.00	27.11	27.11	2
3	32,522	238,615	3,876	(113,018)	238,615	0.00	23.86	23.86	3
4	26,813	211,802	8,918	(119,019)	211,802	0.00	21.18	21.18	4
5	20,445	191,358	12,805	(121,924)	191,358	0.00	191.36	191.36	5
6	13,341	178,016	15,549	(122,470)	178,016	0.00	178.02	178.02	6
7	14,882	163,135	17,161	(121,475)	163,135	0.00	163.13	163.13	7
8	16,599	146,535	18,747	(118,762)	146,535	0.01	1,465.35	1,465.35	8
9	18,516	128,019	20,306	(114,133)	128,019	0.01	1,280.19	1,280.19	9
10	20,653	107,366	21,832	(107,366)	107,366	0.01	1,073.66	1,073.66	10
11	23,037	84,329	37,210	(84,329)	84,329	0.05	4,216.44	4,216.44	11
12	25,697	58,632	36,828	(58,632)	58,632	0.05	2,931.60	2,931.60	12
13	28,663	29,969	36,403	(29,969)	29,969	0.10	2,996.88	2,996.88	13
14	31,972	(2,003)	35,928	2,003	(2,003)	0.10	(200.34)	(200.34)	14
15	35,663	(37,666)	35,399	37,666	(37,666)	0.67	(25,108.40)	(25,108.40)	15

cost of $262. Similar results for the two year, three year and four year plans give average annual per line costs of $178, $142, and $176, respectively. From these figures we see that the plan producing the lowest annual cost per line is the three year plan.

Once the optimal size expansion is determined, the CAPCOST model is run for each of the three components of cost corresponding to access lines, intraswitch busy-hour traffic, and interswitch busy-hour local traffic. Thus, a per-unit annual cost is determined for each variable.

Switch #2, which is losing customers, is treated differently than the others. Here we use a company-wide historical average size expansion as a standard and use the CAPCOST model to solve for the annual cost of this average expansion but with a declining fill rate. For example, suppose the average number of lines added during past switch expansions has been 400. We determine the capital cost and total cost of such expansions from either the models or figures 3-2 and 3-6 as before. These are entered in the CAPCOST model as usual. The fill rate, however, is set to 1 in the first year, and using the data in table 3-16 where 100 lines are lost annually, the fill rate is 300/400 in the second year, 200/400 the third year, 100/400 the fourth year, and 0 the fifth and all successive years. The CAPCOST solution is divided by 400 lines to get the average cost per line. This figure represents the cost per line of having those 400 lines available while they are being placed out of service. The fact that 400 lines was used as the base is perhaps of little significance, but using historical data to determine the base number will certainly pass a reasonableness test.

In the sample calculations, it was assumed that no equipment is salvaged as customers leave the switch. This assumption was made to simplify the example but should not be carried over to a study of actual switches that are losing customers. The reason is that much of the expensive electronic equipment in these switches consists of plug-in modules that can be removed from one switch and plugged into another. The computer aided design system used by Ohio Bell to configure their 1 and 1A ESS switches can provide estimates of the value of salvaged equipment. Such information could be used to adjust the cost of the switches that are losing customers. One may need also to consider the average vintage of the most recent additions to switches that now have negative growth, rather than assume (as we have done here) that the expansion occurs and is fully filled immediately before customers begin leaving the switch. Given the appropriate data, the requisite adjustments to the analysis can be accomplished with the CAPCOST model.

In the case of switch #2, the result for access lines was $5.05; for the intraswitch busy-hour CCS, $1.48 was the solution with an assumed average expansion size of 400 CCS; and the interswitch busy-hour CCS figure was $126.46 with an average expansion size of 2000 CCS. All the average expansion sizes are hypothetical values for illustrative purposes only, since the required historical data have not been scientifically gathered as part of the pilot study. These values are plausible, however.

Table 3-18 summarizes the values obtained from the CAPCOST analyses described above for switches #1 and #2, and it gives hypothetical, but plausible, corresponding values for switch #3.

The final step is to average the cost of each variable across the three switches. This average is a weighted average where the weight for a switch is the proportional amount of the first year's activity in all switches that takes place in the particular switch. An activity is defined to be either a reduction or an increase in the number of units of the variable. If the activities that take place are all independent events, then these weights

TABLE 3-18

THE AVERAGE, ANNUAL, PER-UNIT
COST FOR THREE VARIABLES IN THREE SWITCHES

Variable		Switch		
Symbol	Description	#1	#2	#3
x_1	Access lines	$ 3.24	$ 5.05	$ 1.55
x_2	Interswitch local busy-hour CCS	22.18	126.46	23.90
x_3	Intraswitch busy-hour CCS	1.10	1.48	0.82

Source: Authors' calculations with pilot cost data in switches #1 and #2, and authors' assumed growth data for all switches and assumed model coefficients for switch #3 may be interpreted as the probability that an event occurs in the switch. Thus, the average across switches that can be computed may be thought of as the expected value of the cost of a marginal unit whether it is a marginal increase or a marginal decrease.

For access lines, the total number of events in the first year are 250 + 100 + 400 = 750. For intraswitch busy-hour CCS, the figure is 200 + 50 + 500 = 750, while the interswitch local busy-hour CCS figure is 1200 + 600 + 2000 = 3800. Thus, the expected average incremental cost of the three variables are computed as follows:

An access line costs:

$3.24(250/750) + $5.05(100/750) + $1.55(400/750) = $ 2.58.

An intraswitch busy-hour CCS costs:

$1.10(200/750) + $1.48(50/750) + $0.82(500/750) = $0.94.

An interswitch local busy-hour CCS costs:

$22.18(1200/3800) + $126.46(600/3800) + 23.90(2000/3800) = $39.55.

These figures are used in the next step to compute the cost of a POTS customer.

Step 7: Compute

A POTS customer, whether business or residence, can be viewed as a vector with three components, one for each of the variables of previous steps. For example, (1, 0.6, 1.3) could represent a residential customer and would be interpreted to mean 1 access line, 0.6 intraswitch busy-hour CCS, and 1.3 interswitch local busy-hour CCS. An example of a business customer might be (1, 1.5, 3.4). If we say, for example, that 30% of access line related costs are paid for through state and interstate toll charges, then the two vectors could be modified to reflect only the POTS responsibility thusly: the residential vector becomes (0.7, 0.6, 1.3) and the business vector becomes (0.7, 1.5, 3.4).

The cost of a customer is a straightforward calculation. The costs determined in the previous step can be used as the components of a cost vector. Then a customer cost is the vector multiplication of the customer vector with the cost vector. This results in a residential cost of $53.79 and a business customer cost of $137.69 for this example. These costs exclude a fixed expense item per expansion of $135,000, although that amount was included in the optimization step. Since it is a fixed expense, it does not vary with the amount of capacity for lines or traffic added in an expansion. For the optimal size expansion, this fixed expense item annualizes to $23.13 per line. All or a portion of this cost may be spread to the two POTS customer classes on the basis of lines or some other reasonable factor. For purposes of example, suppose that 70% is to be spread on the basis of lines, and that in the sample switches three quarters of all lines added are expected to go to residential POTS customers while one quarter will go to business POTS customers. This increases the cost for residential customers to $65.93 per year or $5.49 per month. The cost for business customers increases to $141.73 per year or $11.81 per month.

Summary and Findings

A method has been presented that computes, as a final result, the marginal cost of a marginal customer whose service is provided by adding capacity to an existing switch. The method is also applicable, with some modification, to estimating costs for customers whose service is provided by a totally new facility. Such modified estimates would need to include the cost of the central processing unit (CPU) and related equipment. The expansion costs computed in this chapter do not include any CPU costs. The particular problem with the CPU cost is that it is a very lumpy piece of equipment whose cost is probably unrelated to the variables we have used to describe the customer vector. Lumpiness can be handled by averaging over incremental additions of plant, but being unrelated to the variables that describe customers is a fixed-cost problem. Fixed costs will have to be allocated. This study has not addressed the problem of allocating fixed costs other than those arising as part of a switch expansion. Some of the ideas expressed could apply to fixed costs resulting from the installation of a new CPU.

The critical question with respect to our method is whether it is possible to apply it to derive at least part of the marginal cost of a customer. Certainly the computations prescribed in the method are possible if the data needed for the calculations can be obtained. Since the method works with a sample of switches, the question of data availability pertains to each switch in the sample. Since the pilot study involved developing the requisite data for an actual switch, it serves to demonstrate that, indeed, such data can be acquired to perform the specified calculations. The only assumed data for the pilot switch was the forecasted growth in service requirements. Such forecasts are routinely made in the Ohio Bell system and are certainly necessary for every telephone company in order to make informed capacity decisions. Thus, the forecasted data should be readily available and could be provided independently by the PUCO, given available historical data on customer use of telephone services.

A second favorable disclosure of the pilot study was the result that the cost structure for the variables that describe a customer was a very simple structure, making it easy to use. Moreover, it was almost completely separable from the costs for interexchange traffic. The only non-separable part was the fixed expense associated with an expansion but which was also unrelated to the nature of the expansion. While we did suggest methods for allocating the fixed costs, our basic view is that a cost study should generally not allocate costs but should estimate and identify the various components of cost including fixed costs and variable costs. The problem of how all the costs should be recovered is a pricing matter, the solution to which must be based on cost information as well as the objectives being sought by a pricing policy.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

In the first chapter a number of questions were raised. Included among these was what are the appropriate costs and what methods can be used to compute a cost figure for each part of the local network? The first step in developing methods to compute the costs of the different parts of POTS service is to develop a method for one part. This was done for one of the major parts of the system, COE. Procedures have been included to provide some optimization of costs vis-a-vis forecasted service requirements and to annualize these costs. Both of these steps (optimization and annualization) bring the results of the method close to what has been demonstrated to be, on theoretical grounds, the appropriate cost.

What has not been fully addressed as yet is the usefulness of the marginal cost values to solve actual pricing problems. While the long-run marginal cost or its practical equivalent, the average incremental cost, was discussed in chapter 2 where it was concluded that such costs are theoretically the most useful in setting prices, the question remains as to how such costs can be used as a practical matter. Perhaps the best way to answer that is with an example. We quote from a recent monograph by Park and Mitchell who were reporting on an assessment of peak-load pricing for telephone calls:[1]

> Capacity costs will vary with the number of calls (or minutes) that the telephone switching equipment and trunk lines are designed to serve. We are interested[1] in the additional cost of serving one additional call per hour and use the annual equivalent of the incremental investment cost, reflecting such factors as yearly interest, depreciation, maintenance, and taxes.
>
> [1] Because the equipment is available only in "lumps," the cost must actually be estimated as the average incremental cost over some wider range of alternative capacities.

---

[1] Park, Rolla E., and Mitchell, Bridger, M., Optimal Peak-Load Pricing for Local Telephone Calls, The Rand Corporation, Santa Monica, CA (1986), p. 14.

Park and Mitchell could not have performed their analysis of a peak-load pricing policy without the cost figure described by them in the above excerpt. We point out that the type of cost figure that they have described is exactly the type that the method in this report will determine. The actual number that they used came from a 1980 Mountain Bell filing for measured rates in Arizona. In addition, their analysis required usage data and price elasticities which they obtained from the 1976 Illinois experiments conducted by GTE.[2] Park and Mitchell concluded that feasible peak-load pricing schedules would at best result in only modest efficiency gains, and would more likely result in small efficiency losses.

The key point is that if economic efficiency is at least one of the pricing goals, then it is of fundamental importance to acquire the type of cost data obtainable by the method in our report as well as data about usage patterns of the various customer classes and their price elasticities. If there is no goal of economic efficiency, but instead the goal is to simply cover the booked costs, then approximate long-run marginal costs are not needed.

Our recommendation is that the method described in chapter 3 be undertaken with a sample of from 10 to 15 offices in the Ohio Bell territory, and that simultaneously with that effort procedures be found to incorporate O & M expenses and the investment cost of the CPU. We reiterate the recommendation that the 8 point experimental plan be used on these sample offices so that a better understanding can be gained of the cost savings that occur when DID trunks and interoffice traffic capacity are added in the same expansion. The average costs obtained from such a study can be directly applied in the next rate case when combined with usage data and loop costs presently being gathered by Ohio Bell.

---

[2] Cohen, G., "Measured Rates Versus Flat Rates: A Pricing Experiment," presented at the Fifth Annual Telecommunciations Policy Research Conference, Airlie, VA, March 1977.

APPENDIX A
TECHNICAL DISCUSSION

Problem Analysis

This study is a continuation of research begun by Constas.[1] Constas expressed the cost of expanding an existing office as a function of the growth of single lines, DID trunks, intraoffice usage and interoffice local usage. In his model Constas assumed that the total interoffice toll usage in a central office is constant, but recommended that this factor be examined in a future study. The research described in this report follows up on Constas' earlier work and incorporates interoffice toll usage into a model that estimates the capital cost of a capacity expansion in a 1A ESS central office. This appendix presents a detailed description of the experimental design as well as the technical terms and procedures.

Factorial Design

There are five variables that affect the cost of a capacity expansion in a central office.

(1) Growth of single lines
(2) Growth of direct in dial (DID) business trunks
(3) Growth of intraoffice usage
(4) Growth of interoffice local usage
(5) Growth of interoffice toll usage

The traffic measures, items 3, 4 and 5, are in (CCS) hundred call seconds units. The same four sizes, found in the Constas thesis, are used here. There are no structural differences among the four sizes. These four

[1] Constas, Anthonios (1985). Incremental Capital Costs of Local Telephone Service. Masters' thesis, unpublished. Columbus, Ohio: The Ohio State University.

sizes differ only in the size of the various expansion scenarios used to obtain the data for estimating the parameters of the model.

Design of Experiment

1. Factorial design: Factorial designs are widely used in experiments involving several factors where it is necessary to study both the individual and joint effects of these factors on a response. A particular case might involve k factors; each at two levels, "high" and "low." These levels may be quantitative or qualitative. A complete replicate of such a design requires $2*2*...*2 = 2^k$ observations, and is called a 2^k factorial design.

The main effect of a factor may be defined as the change in response produced by the change in the level of the factor. A factor is usually denoted by a capital letter. The high level of a factor in a treatment combination is denoted by the presence of the respective lower case letter, while the low level is denoted by the absence of the corresponding letter.

As an example, suppose that three factors, A, B, and C, each at two levels, are under study. The design is called a 2^3 factorial, and eight treatment combinations are displayed in standard order as (1), a, b, ab, c, ac, bc, and abc. An observation taken under the high level of A and C and the low level of B is represented by ac. The observations of a 2^3 factorial design that involve quantitative factors may be equivalently described by the following linear regression model:

$$Y = \beta_0 - \beta_1 x_1 + \beta_2 x_2 - \beta_3 x_3 - \beta_4 x_1 x_2 - \beta_5 x_1 x_3 - \beta_6 x_2 x_3 - \beta_7 x_1 x_2 x_3$$

where β_0 is the overall average effect, β_1, β_2 and β_3 are the main effects, and β_4, β_5, β_6 and β_7 are interaction effects.

As the number of factors increase in an experiment, the number of runs required for a complete replicate of the design rapidly outgrows the resources of most experimenters. In this research there are five factors.

Hence, a complete replicate of a 2^5 experiment requires 32 runs on COEES[2]. Because of high cost associated with running the COEES program 32 times, the idea of a one-half fraction of 2^5 is introduced.

If we can assume that certain high-order interactions are negligible, then the information on main effects and lower-order interactions may be obtained by running only a fraction of the complete factorial experiment. Thus, obtaining a reduction in cost without sacrificing accuracy.

2. The one-half fraction of the 2^k design: Consider the same three-factor example discussed earlier. Let us say the experimenter cannot afford to run $2^3 = 8$ treatment combinations. He can, however, afford 4 runs so this suggests a one-half fraction of a 2^3 design. Because the design contains 2^{3-1} or 4 treatment combinations, a one-half fraction of a 2^3 design is called a 2^{3-1} design. A high level of a factor is denoted by a plus sign (+) and a low level by a minus sign (-) in table A-1 below, and illustrates a full 2^3 factorial design.

TABLE A-1

A FULL 2^3 FACTORIAL DESIGN

Treatment Combination	I	A	B	C	ABC
(1)	+	-	-	-	-
a	+	+	-	-	+
b	+	-	+	-	+
ab	+	+	+	-	-
c	+	-	-	+	+
ac	+	+	-	+	-
bc	+	-	+	+	-
abc	+	+	+	+	+

[2] COEES is the Central Office Equipment Engineering System, a computerized model that telephone companies use for designing new central offices, or for planning facility additions to existing offices.

The 2^{3-1} design is formed by selecting only those treatment combinations that yield a plus sign on the ABC effect. The ABC term is called the generator of this particular fraction. Furthermore, the identity element I is always plus, so I = ABC is called the defining relation for the design.

To avoid making 32 runs on the COEES, a 2^{5-1} design may be used. This implies that the number of runs has decreased from 32 to 16. To decide which 16 out of the 32 treatment combinations to select, the following defining relation for the design is chosen:

I = ABCDE

where

A = Single lines added
B = DID trunks added
C = Intraoffice usage added
D = Interoffice local usage added
E = Interoffice toll usage added

The selection criterion is based on selecting the 16 scenarios (treatment combinations) that yield a plus sign on the ABCDE effect (see tables A-2, A-3).

TABLE A-2

FULL FACTORIAL DESIGN

Treatment Combination	I	A	B	C	D	E	ABCDE
(1)	+	-	-	-	-	-	-
a	+	+	-	-	-	-	+
b	+	-	+	-	-	-	+
ab	+	+	+	-	-	-	-
c	+	-	-	+	-	-	+
ac	+	+	-	+	-	-	-
bc	+	-	+	+	-	-	-
abc	+	+	+	+	-	-	+
d	+	-	-	-	+	-	+
ad	+	+	-	-	+	-	-
bd	+	-	+	-	+	-	-
abd	+	+	+	-	+	-	+
cd	+	-	-	+	+	-	-
acd	+	+	-	+	+	-	+
bcd	+	-	+	+	+	-	+
abcd	+	+	+	+	+	-	-
e	+	-	-	-	-	+	+
ae	+	+	-	-	-	+	-
be	+	-	+	-	-	+	-
abe	+	+	+	-	-	+	+
ce	+	-	-	+	-	+	-
ace	+	+	-	+	-	+	+
bce	+	-	+	+	-	+	+
abce	+	+	+	+	-	+	-
de	+	-	-	-	+	+	-
ade	+	+	-	-	+	+	+
bde	+	-	+	-	+	+	+
abde	+	+	+	-	+	+	-
cde	+	-	-	+	+	+	+
acde	+	+	-	+	+	+	-
bcde	+	-	+	+	+	+	-
abcde	+	+	+	+	+	+	+

TABLE A-3

HALF-FRACTION FACTORIAL DESIGN

Scenario Number	Treatment Combination
1	e
2	a
3	b
4	abe
5	c
6	ace
7	bce
8	abc
9	d
10	ade
11	bde
12	abd
13	cde
14	acd
15	bcd
16	abcde

These scenarios have been selected according to the generator I=ABCDE (all treatment combinations that yield a plus sign on the ABCDE effect in table A-2).

3. Vector Representation of a Scenario: Consider the following row vector:

S = (350, 90, 300, 1800, 400)

From the experiment, for example, scenario 3 is treatment combination b (table A-3); this is equivalent to saying that only factor B is at high level. The b treatment combination can be represented by vector S1, S2, S3 or S4 depending on what size of expansion is taken, where

S1 = (0, 90, 0, 0, 0) , if size 1 is chosen
S2 = S1 + S , if size 2 is chosen
S3 = S1 + 2*S , if size 3 is chosen
S4 = S1 + 3*S , if size 4 is chosen

Similar analyses were done to find the vector representation of all 15 remaining scenarios in the four different sizes. It shall be noted here that one run of COEES will produce a solution for a scenario in all four size expansions.

Regression Analysis

Data Request

A data request was sent to Ohio Bell in November of 1985 (exhibit 1, found at the end of this appendix). This request contained the sixteen scenarios (treatment combinations) that the one-half fraction design selected. Each scenario was made up of eight alternatives. These alternatives depended on the size of expansion that might be decided in a specific year. In addition, each alternative had up to four possible jobs. A job is an actual task of adding capacity in a given year. Since the study was to examine the cost of an expansion made now, rather than an expansion made a year or more from now, only the first job (job 1) was considered.

The output of each of the 16 runs contained an equipment list for every alternative and its corresponding costs (capital, expense, removal, grand total). An equipment list itemized the amount and type of equipment present before the expansion, and the number and type added or removed to satisfy a given scenario.

Because of the balance in the experiment achieved with factorial designs, the data are arranged in a special form. This form helps in searching for patterns in the data that are useful in error checking and for explaining discrete jumps in capital costs due to lumpiness. Four tables are formed since four sizes of expansion are possible (tables 3-1, 3-2, 3-3 and 3-4 in chapter 3). To construct each table, all 16 scenarios are listed as column headings and an exhaustive list of all equipment as row headings. The data from the 16 output runs for the four tables are summarized in table A-4.

Mathematical Model

In this project, four 2^{5-1} designs are considered to estimate the capital cost of a capacity expansion in a central office. The five variables affecting the cost are the following:

A = Single lines added

B = DID trunks added

C = Intraoffice usage added

D = Interoffice local usage added

E = Interoffice toll usage added

In every size, each variable is used at two levels, "high" and "low" (table A-5).

TABLE A-4

ALTERNATIVE AND JOB SELECTION

Treatment Combination	Scenario Number	Size 1	Size 2	Size 3	Size 4
e	1	Alternative 1	Alternative 5	Alternative 6	Alternative 8
a	2	Alternative 1	Alternative 5	Alternative 6	Alternative 8
b	3	Alternative 1	Alternative 5	Alternative 6	Alternative 8
abe	4	Alternative 1	Alternative 5	Alternative 6	Alternative 8
c	5	Alternative 1	Alternative 5	Alternative 6	Alternative 8
ace	6	Alternative 1	Alternative 5	Alternative 6	Alternative 8
bce	7	Alternative 1	Alternative 5	Alternative 6	Alternative 8
abc	8	Alternative 1	Alternative 5	Alternative 6	Alternative 8
d	9	Alternative 1	Alternative 5	Alternative 6	Alternative 8
ade	10	Alternative 1	Alternative 5	Alternative 6	Alternative 8
bde	11	Alternative 1	Alternative 5	Alternative 6	Alternative 8
abd	12	Alternative 1	Alternative 5	Alternative 6	Alternative 8
cde	13	Alternative 1	Alternative 5	Alternative 6	Alternative 8
acd	14	Alternative 1	Alternative 5	Alternative 6	Alternative 8
bcd	15	Alternative 1	Alternative 5	Alternative 6	Alternative 8
abcde	16	Alternative 1	Alternative 5	Alternative 6	Alternative 8

Note:

All alternatives are taken under job 1 since the study was to examine the cost of an expansion made now rather than a year or more from now.

For example, to find the amount of equipment added or removed and the associated costs that satisfy the b treatment combination in size 4, you look in the output of scenario 3, alternative 8, job 1.

TABLE A-5

RANGES OF VARIABLES

	Size 1	Size 2	Size 3	Size 4
A	[0,350]	[350,700]	[700,1050]	[1050,1400]
B	[0,90]	[90,180]	[180,270]	[270,360]
C	[0,300]	[300,600]	[600,900]	[900,1200]
D	[0,1800]	[1800,3600]	[3600,5400]	[5400,7200]
E	[0,400]	[400,800]	[800,1200]	[1200,1600]

There are a total of 64 observations (with 16 for each size) in this study. A decision has to be made whether to fit all 64 data points to one regression model, a composite model, or to consider four regression models; for each size it is necessary to come up with a model that best fits 16 data points. The latter is chosen because its cost estimates are significantly closer to the actual cost figures. Tables 3-9, 3-10, 3-11 and 3-12 show the four sets of data that are used to fit the four cost models.

To fit a regression model, a general model had to be assumed. The design was a fractional one and this led to the assumption that the fifth order interaction effect was negligible. The only interaction effect that was large enough to be included in the model was the BD interaction; a change in the cost was clear if both DID trunks and inter-office local usage were added at the same time.

The general regression model finally assumed for all four sizes was the following:

$$Yc(i) = \beta_0 + \beta_1 A + \beta_2 B + \beta_3 C + \beta_4 D + \beta_5 E + \beta_6 BD$$

for i = 1,2,3,4

where

Yc (i) is the capital cost of size i expansion.

A, B, C, D and E defined earlier.

BD is the interaction effect between DID trunks and interoffice local usage.

EXHIBIT 1
OF APPENDIX A

2^5 FRACTIONAL FACTORIAL EXPERIMENTAL CONSTRUCTION PLAN
PROJECT NAME: MARGINAL COST OF PLAIN OLD TELEPHONE SERVICE (POTS)

by

Clark A. Mount-Campbell
Associate Professor of Industrial and
Systems Engineering

Baumin Lee
Graduate Research Associate

THE NATIONAL REGULATORY RESEARCH INSTITUTE
Columbus, Ohio

November, 1985

SCENARIO 1

TABLE 1A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 1

TABLE 1B

Variables	Busy Season	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 1B continued

Variables	Busy Season	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 2

TABLE 2A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 2

TABLE 2B

Variables		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		0	350	350	350	0	350	700	-	0	1050	-	-	0	350	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 2B continued

Variables		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	4000	-	400	400	1200	-	-	-

SCENARIO 3

TABLE 3A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 3

TABLE 3B

Variables	Busy Season	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 3B continued

Variables	Busy Season	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 4

TABLE 4A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 4

TABLE 4B

Variables	Busy Season	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 4B continued

Variables	Busy Season	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 5

TABLE 5A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 5

TABLE 5B

Variables	Busy Season	Alternative 1: 1	Alternative 1: 2	Alternative 1: 3	Alternative 1: 4	Alternative 2: 1	Alternative 2: 2	Alternative 2: 3	Alternative 2: 4	Alternative 3: 1	Alternative 3: 2	Alternative 3: 3	Alternative 3: 4	Alternative 4: 1	Alternative 4: 2	Alternative 4: 3	Alternative 4: 4
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 5B continued

Variables	Busy Season	Alternative 5: 1	Alternative 5: 2	Alternative 5: 3	Alternative 5: 4	Alternative 6: 1	Alternative 6: 2	Alternative 6: 3	Alternative 6: 4	Alternative 7: 1	Alternative 7: 2	Alternative 7: 3	Alternative 7: 4	Alternative 8: 1	Alternative 8: 2	Alternative 8: 3	Alternative 8: 4
Growth of single lines		700	-	700	-	700	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 6

TABLE 6A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 6

TABLE 6B

Variables		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 6B continued

Variables		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 7

TABLE 7A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 7

TABLE 7B

Variables	Busy Season	Alt. 1: 1	Alt. 1: 2	Alt. 1: 3	Alt. 1: 4	Alt. 2: 1	Alt. 2: 2	Alt. 2: 3	Alt. 2: 4	Alt. 3: 1	Alt. 3: 2	Alt. 3: 3	Alt. 3: 4	Alt. 4: 1	Alt. 4: 2	Alt. 4: 3	Alt. 4: 4
		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 7B continued

Variables	Busy Season	Alt. 5: 1	Alt. 5: 2	Alt. 5: 3	Alt. 5: 4	Alt. 6: 1	Alt. 6: 2	Alt. 6: 3	Alt. 6: 4	Alt. 7: 1	Alt. 7: 2	Alt. 7: 3	Alt. 7: 4	Alt. 8: 1	Alt. 8: 2	Alt. 8: 3	Alt. 8: 4
		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 8

TABLE 8A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 8

TABLE 8B

Variables	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines	0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk	0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 8B continued

Variables	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines	350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk	90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage (CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage (CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic (CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 9

TABLE 9A

Variables \ Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 9

TABLE 9B

Variables	Busy Season	Alt. 1: 1	Alt. 1: 2	Alt. 1: 3	Alt. 1: 4	Alt. 2: 1	Alt. 2: 2	Alt. 2: 3	Alt. 2: 4	Alt. 3: 1	Alt. 3: 2	Alt. 3: 3	Alt. 3: 4	Alt. 4: 1	Alt. 4: 2	Alt. 4: 3	Alt. 4: 4
Growth of single lines		350	350	35[illegible]	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 9B continued

Variables	Busy Season	Alt. 5: 1	Alt. 5: 2	Alt. 5: 3	Alt. 5: 4	Alt. 6: 1	Alt. 6: 2	Alt. 6: 3	Alt. 6: 4	Alt. 7: 1	Alt. 7: 2	Alt. 7: 3	Alt. 7: 4	Alt. 8: 1	Alt. 8: 2	Alt. 8: 3	Alt. 8: 4
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 10

TABLE 10A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 10

TABLE 10B

Variables		Busy Season	Alternative 1: 1	2	3	4	Alternative 2: 1	2	3	4	Alternative 3: 1	2	3	4	Alternative 4: 1	2	3	4
Growth of single lines			0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk			90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)		0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)		1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)		400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 10B continued

Variables		Busy Season	Alternative 5: 1	2	3	4	Alternative 6: 1	2	3	4	Alternative 7: 1	2	3	4	Alternative 8: 1	2	3	4
Growth of single lines			350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk			180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)		300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)		3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)		800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 11

TABLE 11A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 11

TABLE 11B

Variables		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 11B continued

Variables		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 12

TABLE 12A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 12

TABLE 12B

Variables		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 12B continued

Variables		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 13

TABLE 13A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 13

TABLE 13B

Variables		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 13B continued

Variables		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Busy Season	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 14

TABLE 14A

Variables / Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	0	400	400	400

SCENARIO 14

TABLE 14B

Variables	Busy Season	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	0	400	400	400	0	400	800	-	0	1200	-	-	0	800	-	400

TABLE 14B continued

Variables	Busy Season	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)	400	-	800	-	800	-	-	400	400	-	400	400	1200	-	-	-

SCENARIO 15

TABLE 15A

Variables / Busy Season	1	2	3	4
Growth of single lines	350	350	350	350
Growth of DID trunk	90	90	90	90
Growth of intraoffice busy hour usage (CCS)	300	300	300	300
Growth of interoffice busy hour local usage (CCS)	1800	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 15

TABLE 15B

Variables	Busy Season	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
Growth of DID trunk		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
Growth of intraoffice busy hour usage	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
Growth of interoffice busy hour local usage	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	3600	-	-	1800	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 15B continued

Variables	Busy Season	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Growth of single lines		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
Growth of DID trunk		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
Growth of intraoffice busy hour usage	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
Growth of interoffice busy hour local usage	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

SCENARIO 16

TABLE 16A

Variables \ Busy Season	1	2	3	4
Growth of single lines	0	350	350	350
Growth of DID trunk	0	90	90	90
Growth of intraoffice busy hour usage (CCS)	0	300	300	300
Growth of interoffice busy hour local usage (CCS)	0	1800	1800	1800
Growth of interexchange busy hour traffic (CCS)	400	400	400	400

SCENARIO 16

TABLE 16B

Variables	Busy Season	Alternative 1: 1	2	3	4	Alternative 2: 1	2	3	4	Alternative 3: 1	2	3	4	Alternative 4: 1	2	3	4
Growth of single lines		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
Growth of DID trunk		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
Growth of intraoffice busy hour usage	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
Growth of interoffice busy hour local usage	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800
Growth of interexchange busy hour traffic	(CCS)	400	400	400	400	400	400	800	-	400	1200	-	-	400	800	-	400

TABLE 16B continued

Variables	Busy Season	Alternative 5: 1	2	3	4	Alternative 6: 1	2	3	4	Alternative 7: 1	2	3	4	Alternative 8: 1	2	3	4
Growth of single lines		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
Growth of DID trunk		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
Growth of intraoffice busy hour usage	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
Growth of interoffice busy hour local usage	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-
Growth of interexchange busy hour traffic	(CCS)	800	-	800	-	1200	-	-	400	800	-	400	400	1600	-	-	-

APPENDIX B

2^{4-1} FRACTIONAL FACTORIAL EXPERIMENTAL CONSTRUCTION PLAN
PROJECT NAME: MARGINAL COST OF PLAIN OLD TELEPHONE SERVICE (POTS)

by

Clark A. Mount-Campbell
Associate Professor of Industrial and
Systems Engineering

Hisham M. Chouoiki
Graduate Research Associate

THE NATIONAL REGULATORY RESEARCH INSTITUTE
Columbus, Ohio

September, 1986

In a 2^{4-1} fractional factorial experimental design, eight observations (treatment combinations or scenarios) are required. These observations are selected from a full 2^4 factorial design according to the defining equation I = ABCD. The eight observations (scenarios) are listed in the tables of this appendix that are labeled xA where x is the scenario number.

Eight four-year construction programs are possible to satisfy the demand requirements specified in a given scenario. These are given in tables with labels xB where again, x is the scenario number. For example, the eight alternatives specified for scenario 1 are listed in table 1B. Similarly, the alternatives for scenarios 2, 3, 4, 5, 6, 7 and 8 are listed in tables 2B to 8B, respectively.

The tables of this appendix are given in the same format as the actual data request shown at the end of appendix A in the exhibit 1.

SCENARIO 1

TABLE 1A

Variables / Year	1	2	3	4
Single lines added	0	350	350	350
DID Trunks added	0	90	90	90
Intraoffice usage added (CCS)	0	300	300	300
Interoffice local added (CCS)	0	1800	1800	1800

SCENARIO 1

TABLE 1B

Variables	Year	Alternative 1: 1	2	3	4	Alternative 2: 1	2	3	4	Alternative 3: 1	2	3	4	Alternative 4: 1	2	3	4
SINGLE LINES ADDED		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
DID TRUNKS ADDED		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 1B continued

Variables	Year	Alternative 5: 1	2	3	4	Alternative 6: 1	2	3	4	Alternative 7: 1	2	3	4	Alternative 8: 1	2	3	4
SINGLE LINES ADDED		350	-	700	-	700	-	-	350	350	-	300	350	1050	-	-	-
DID TRUNKS ADDED		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	1800	-	3600	-	3600	-	-	1300	1800	-	1800	1800	5400	-	-	-

SCENARIO 2

TABLE 2A

Variables / Year	1	2	3	4
Single lines added	350	350	350	350
DID trunks added	0	90	90	90
Intraoffice usage added (CCS)	0	300	300	300
Interoffice local added (CCS)	1800	1800	1800	1800

SCENARIO 2

TABLE 2B

Variables		Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Year	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED		45	90	90	90	45	90	180	-	45	270	-	-	45	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 2B continued

Variables		Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Year	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED		135	-	160	-	225	-	-	90	135	-	90	90	315	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-

SCENARIO 3

TABLE 3A

Variables / Year	1	2	3	4
Single lines added	0	350	350	350
DID trunks added	90	90	90	90
Intraoffice usage added (CCS)	0	300	300	300
Interoffice local added (CCS)	1800	1800	1800	1800

SCENARIO 3

TABLE 3B

Variables	Alt. 1 Yr 1	Alt. 1 Yr 2	Alt. 1 Yr 3	Alt. 1 Yr 4	Alt. 2 Yr 1	Alt. 2 Yr 2	Alt. 2 Yr 3	Alt. 2 Yr 4	Alt. 3 Yr 1	Alt. 3 Yr 2	Alt. 3 Yr 3	Alt. 3 Yr 4	Alt. 4 Yr 1	Alt. 4 Yr 2	Alt. 4 Yr 3	Alt. 4 Yr 4
SINGLE LINES ADDED	350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED	45	90	90	90	45	90	180	-	45	270	-	-	45	180	-	90
INTRAOFFICE USAGE ADDED (CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
INTEROFFICE USAGE ADDED (CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800

TABLE 3B continued

Variables	Alt. 5 Yr 1	Alt. 5 Yr 2	Alt. 5 Yr 3	Alt. 5 Yr 4	Alt. 6 Yr 1	Alt. 6 Yr 2	Alt. 6 Yr 3	Alt. 6 Yr 4	Alt. 7 Yr 1	Alt. 7 Yr 2	Alt. 7 Yr 3	Alt. 7 Yr 4	Alt. 8 Yr 1	Alt. 8 Yr 2	Alt. 8 Yr 3	Alt. 8 Yr 4
SINGLE LINES ADDED	700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED	155	-	190	-	225	-	-	90	135	-	90	90	315	-	-	-
INTRAOFFICE USAGE ADDED (CCS)	300	-	600	-	600	-	-	300	800	-	300	300	900	-	-	-
INTEROFFICE USAGE ADDED (CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-

SCENARIO 4

TABLE 4A

Variables / Year	1	2	3	4
Single lines added	350	350	350	350
DID trunks added	90	90	90	90
Intraoffice usage added (CCS)	0	300	300	300
Interoffice local added (CCS)	0	1800	1800	1800

SCENARIO 4

TABLE 4B

Variables	Year	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 4B continued

Variables	Year	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED		180	-	180	-	270	-	-	90	180	-	90	90	300	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-

SCENARIO 5

TABLE 5A

Variables / Year	1	2	3	4
Single lines added	0	350	350	350
DID trunks added	0	90	90	90
Intraoffice usage added (CCS)	300	300	300	300
Interoffice local added (CCS)	1800	1800	1800	1800

SCENARIO 5
TABLE 5B

Variables	Year	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
DID TRUNKS ADDED		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800

TABLE 5B continued

Variables	Year	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
DID TRUNKS ADDED		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-

SCENARIO 6

TABLE 6A

Variables / Year	1	2	3	4
Single lines added	350	350	350	350
DID trunks added	0	90	90	90
Intraoffice usage added (CCS)	300	300	300	300
Interoffice local added (CCS)	0	1800	1800	1800

SCENARIO 6
TABLE 6B

Variables	Year	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED		0	90	90	90	0	90	180	-	0	270	-	-	0	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 6B continued

Variables	Year	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED		90	-	180	-	180	-	-	90	90	-	90	90	270	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-

SCENARIO 7

TABLE 7A

Variables / Year	1	2	3	4
Single lines added	0	350	350	350
DID trunks added	90	90	90	90
Intraoffice usage added (CCS)	300	300	300	300
Interoffice local added (CCS)	0	1800	1800	1800

SCENARIO 7

TABLE 7B

Variables / Year	Alt. 1: 1	Alt. 1: 2	Alt. 1: 3	Alt. 1: 4	Alt. 2: 1	Alt. 2: 2	Alt. 2: 3	Alt. 2: 4	Alt. 3: 1	Alt. 3: 2	Alt. 3: 3	Alt. 3: 4	Alt. 4: 1	Alt. 4: 2	Alt. 4: 3	Alt. 4: 4
SINGLE LINES ADDED	0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
DID TRUNKS ADDED	90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
INTRAOFFICE USAGE ADDED (CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
INTEROFFICE USAGE ADDED (CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 7B continued

Variables / Year	Alt. 5: 1	Alt. 5: 2	Alt. 5: 3	Alt. 5: 4	Alt. 6: 1	Alt. 6: 2	Alt. 6: 3	Alt. 6: 4	Alt. 7: 1	Alt. 7: 2	Alt. 7: 3	Alt. 7: 4	Alt. 8: 1	Alt. 8: 2	Alt. 8: 3	Alt. 8: 4
SINGLE LINES ADDED	350	-	700	-	700	-	-	350	350	-	350	350	1800	-	-	-
DID TRUNKS ADDED	180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
INTRAOFFICE USAGE ADDED (CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1800	-	-	-
INTEROFFICE USAGE ADDED (CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-

SCENARIO 8

TABLE 8A

Variables / Year	1	2	3	4
Single lines added	350	350	350	350
DID trunks added	90	90	90	90
Intraoffice usage added (CCS)	300	300	300	300
Interoffice local added (CCS)	1800	1800	1800	1800

SCENARIO 8

TABLE 8B

Variables	Year	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED		90	90	90	90	90	90	180	-	90	270	-	-	90	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800

TABLE 8B continued

Variables	Year	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED		180	-	180	-	270	-	-	90	180	-	90	90	360	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1800	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-

APPENDIX C

2^{3-1} FRACTIONAL FACTORIAL EXPERIMENTAL CONSTRUCTION PLAN
PROJECT NAME: MARGINAL COST OF PLAIN OLD TELEPHONE SERVICE (POTS)

by

Clark A. Mount-Campbell
Associate Professor of Industrial and
Systems Engineering

Hisham M. Chouoiki
Graduate Research Associate

THE NATIONAL REGULATORY RESEARCH INSTITUTE
Columbus, Ohio

September, 1986

In a 2^{3-1} fractional factorial experimental design, four observations (treatment combinations or scenarios) are required. These observations are selected from a full 2^3 factorial design according to the defining equation I = ACD. The four observations (scenarios) are listed in the tables of this appendix that are labeled xA where x is the scenario number. Factor B is held fixed in all scenarios thereby reducing the number of factors to 3.

Eight four-year construction programs are possible to satisfy the demand requirements specified in a given scenario. These are given in tables with labels xB where again, x is the scenario number. For example, the eight alternatives specified for scenario 1 are listed in table 1B. Similarly, the alternatives for scenarios 2, 3 and 4 are listed in tables 2B to 4B, respectively.

The tables in this appendix are given in the same format as the actual data request shown at the end of appendix A in the exhibit 1.

SCENARIO 1

TABLE 1A

Variables / Year	1	2	3	4
Single lines added	0	350	350	350
DID trunks added	45	90	90	90
Intraoffice usage added (CCS)	0	300	300	300
Interoffice local added (CCS)	0	1800	1800	1800

SCENARIO 1

TABLE 1B

Variables	Year	Alternative 1: 1	2	3	4	Alternative 2: 1	2	3	4	Alternative 3: 1	2	3	4	Alternative 4: 1	2	3	4
SINGLE LINES ADDED		0	350	350	350	0	350	700	-	0	1050	-	-	0	700	-	350
DID TRUNKS ADDED		45	90	90	90	0	90	180	-	45	270	-	-	45	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 1B continued

Variables	Year	Alternative 5: 1	2	3	4	Alternative 6: 1	2	3	4	Alternative 7: 1	2	3	4	Alternative 8: 1	2	3	4
SINGLE LINES ADDED		350	-	700	-	700	-	-	350	350	-	300	350	1050	-	-	-
DID TRUNKS ADDED		135	-	180	-	225	-	-	90	135	-	90	90	315	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	300	-	600	-	600	-	-	300	300	-	300	300	900	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-

SCENARIO 1

TABLE 2A

Variables / Year	1	2	3	4
Single lines added	350	350	350	350
DID trunks added	45	90	90	90
Intraoffice usage added (CCS)	300	300	300	300
Interoffice local added (CCS)	0	1800	1800	1800

SCENARIO 2

TABLE 2B

Variables	Unit	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
	Year	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED		45	90	90	90	45	90	180	-	45	270	-	-	45	180	-	90
INTRAOFFICE USAGE ADDED	(CCS)	300	300	300	300	300	300	600	-	300	900	-	-	300	600	-	300
INTEROFFICE USAGE ADDED	(CCS)	0	1800	1800	1800	0	1800	3600	-	0	5400	-	-	0	3600	-	1800

TABLE 2B continued

Variables	Unit	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
	Year	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED		700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED		135	-	180	-	225	-	-	90	135	-	90	90	315	-	-	-
INTRAOFFICE USAGE ADDED	(CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
INTEROFFICE USAGE ADDED	(CCS)	1800	-	3600	-	3600	-	-	1800	1800	-	1800	1800	5400	-	-	-

SCENARIO 3

TABLE 3A

Variables / Year	1	2	3	4
Single lines added	350	350	350	350
DID trunks added	45	90	90	90
Intraoffice usage added (CCS)	0	300	300	300
Interoffice local added (CCS)	1800	1800	1800	1800

SCENARIO 3

TABLE 3B

Variables	Alternative 1				Alternative 2				Alternative 3				Alternative 4			
Year	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED	350	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED	45	90	90	90	45	90	180	-	45	270	-	-	45	180	-	90
INTRAOFFICE USAGE ADDED (CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
INTEROFFICE USAGE ADDED (CCS)	1800	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800

TABLE 3B continued

Variables	Alternative 5				Alternative 6				Alternative 7				Alternative 8			
Year	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
SINGLE LINES ADDED	700	-	700	-	1050	-	-	350	700	-	350	350	1400	-	-	-
DID TRUNKS ADDED	135	-	190	-	225	-	-	90	135	-	90	90	315	-	-	-
INTRAOFFICE USAGE ADDED (CCS)	300	-	600	-	600	-	-	300	800	-	300	300	900	-	-	-
INTEROFFICE USAGE ADDED (CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-

SCENARIO 4

TABLE 4A

Variables / Year	1	2	3	4
Single lines added	0	350	350	350
DID trunks added	45	90	90	90
Intraoffice usage added (CCS)	300	300	300	300
Interoffice local added (CCS)	1800	1800	1800	1800

SCENARIO 4

TABLE 4B

Variables / Year	Alternative 1: 1	2	3	4	Alternative 2: 1	2	3	4	Alternative 3: 1	2	3	4	Alternative 4: 1	2	3	4
SINGLE LINES ADDED	0	350	350	350	350	350	700	-	350	1050	-	-	350	700	-	350
DID TRUNKS ADDED	45	90	90	90	45	90	180	-	45	270	-	-	45	180	-	90
INTRAOFFICE USAGE ADDED (CCS)	0	300	300	300	0	300	600	-	0	900	-	-	0	600	-	300
INTEROFFICE USAGE ADDED (CCS)	0	1800	1800	1800	1800	1800	3600	-	1800	5400	-	-	1800	3600	-	1800

TABLE 4B continued

Variables / Year	Alternative 5: 1	2	3	4	Alternative 6: 1	2	3	4	Alternative 7: 1	2	3	4	Alternative 8: 1	2	3	4
SINGLE LINES ADDED	350	-	700	-	700	-	-	350	350	-	350	350	1050	-	-	-
DID TRUNKS ADDED	135	-	180	-	225	-	-	90	135	-	90	90	315	-	-	-
INTRAOFFICE USAGE ADDED (CCS)	600	-	600	-	900	-	-	300	600	-	300	300	1200	-	-	-
INTEROFFICE USAGE ADDED (CCS)	3600	-	3600	-	5400	-	-	1800	3600	-	1800	1800	7200	-	-	-

APPENDIX D
The CAPCOST PROGRAM

Program Description

This program is written with Lotus 1-2-3, version 1.0, and makes extensive use of macros. The main objective of the program is to compute the annual equivalent cost of an investment in capital equipment. The program includes a standard engineering economy model and a regulatory model combined into one to facilitate comparison of the two approaches. Both models incorporate survival curves and fill rates. As may be seen, the models obtain identical results in terms of the annual equivalent cost of an investment.

Program Contents

The spreadsheet portion of the program is made up of two parts. The first part provides space for input data, while the second is the main output of the program. Some computed results are also contained in the first part.

The first part contains three sections. The first section contains company-specific information such as the capital structure and the composite income tax rate. The second section includes investment-specific information such as the time horizon, regulatory book life, tax life, initial investment, O&M costs and the three Gompretz-Makeham parameters which are presently used as the survival curve (Note: Any survival curve may be used but the corresponding probabilities must be manually added as input). The third section contains information about the solution to the problem of finding the annual equivalent cost.

The specific cells for the three sections are defined as follows: (Note: Capital letters denote spreadsheet columns and numbers denote spreadsheet rows.)

Cell description for section 1:

E3 = Percentage composed of debt
E4 = Average interest rate on debt
E5 = Percentage composed of equity
E6 = Average rate of return on equity
E7 = Composite cost of capital
E8 = Composite income tax rate

Cell description for section 2:

I2 = Time horizon
I3 = Initial amount invested
I4 = Regulatory book life
I5 = Tax life
I6 = Method of tax depreciation used
1) Sum-of-years digits (SOYD)
2) Straight line (SL)
3) Double declining balance that switches to straight line (DDB/SL)
4) Accelerated cost recovery system (ACRS)

I7 = O&M growth rate
I8 = O&M gradient change
I9 = O&M initial cost

Gompertz-Makeham parameters:

I11 = s = 0.99
I12 = g = 0.955
I13 = c = 1.4

Cell description for section 3:

D10 = Lower bound of the interval that contains the solution
D12 = Upper bound of the interval that contains the solution
D11 = Midpoint of the interval
D13 = Width of the interval
E11 = Expected net present value of unrecovered investment
E13 = The final interval width required

The second part of the spreadsheet gives the year-by-year computations of the following: (Note: Capital letters denote spreadsheet columns)

A = K = L = U = V = AE = Year number

B = Fill rate

C = Revenues

D = Special expenses that may be incurred in any given year

E = Operation and maintenance (O&M) cost

F = Book depreciation

G = Accumulated book depreciation

H = Tax depreciation

I = Deferred taxes

J = Accumulated tax deferrals

M = Rate base

N = Actual interest on debt

O = Allowed interest on debt

P = Taxable income

Q = Income

R = After tax cash flow (ATCF)

S = Actual equity income

T = Allowed equity income

W = Capital consumption (economic depreciation)

X = Unrecovered investment

Y = Regulatory surplus (difference between revenues and revenue requirement)

Z = Accumulated future worth of surplus

AA = Unrecovered regulated investment

AB = Probability of retirement in a year

AC = Actual probability weighted unrecovered investment

AD = Regulated probability weighted unrecovered investment

Formulas

The formulas needed to calculate the year-by-year figures are given below. The symbols A(t), B(t), etc., are used to denote the value in column A, B, etc., found in the row corresponding to year t, where t varies between 1 and I2 (time horizon).

A(t) = t

B(t) = 1 (Default value--The user may alter the value in any or all years)

C(t) = Constant revenue, for every t

D(t) = 0 (Default value--the user may alter the value in any or all years)

$$E(t) = \begin{cases} I9, & t=1 \\ I8+(1+I7)*E(t-1), & t>1 \end{cases}$$

F(t) = SL depreciation

G(t) = F(t)+G(t-1)

$$H(t) = \begin{cases} [2*(I5+1-t)/(I5^{\wedge}2+I5)]*I3, & \text{if SOYD} \\ I3/I5, & \text{if SL} \\ 2*[(I3*(1-2/I5)^{\wedge}(t-1))/I5], & \text{if DDB/SL (DDB part)} \\ I3/I5, & \text{if DDB/SL (SL part)} \\ \text{constant}*I3, & \text{if ACRS} \end{cases}$$

$$I(t) = [H(t)-F(t)]*E8$$

$$J(t) = J(t-1)+I(t)$$

$$M(t) = I3-G(t)-J(t)$$

$$N(t) = \begin{cases} E3*E4*I3, & t=1 \\ E3*E4*X(t-1), & t>1 \end{cases}$$

$$O(t) = \begin{cases} E3*E4*I3, & t=1 \\ E3*E4*M(t-1), & t>1 \end{cases}$$

$$P(t) = C(t)-H(t)-N(t)-D(t)-E(t)$$

$$Q(t) = E8*P(t)$$

$$R(t) = C(t)-D(t)-E(t)-Q(t)$$

$$S(t) = \begin{cases} E5*E6*I3, & t=1 \\ E5*E6*X(t-1), & t>1 \end{cases}$$

$$T(t) = \begin{matrix} E5*E6*I3, & t=1 \\ E5*E6*M(t-1), & t>1 \end{matrix}$$

$$W(t) = R(t)-N(t)-S(t)$$

$$X(t) = \begin{cases} I3 - W(t), & t=1 \\ X(t-1) - W(t), & t>1 \end{cases}$$

$$Y(t) = C(t)-D(t)-E(t)-I(t)-O(t)-Q(t)-T(t)-F(t)$$

$$Z(t) = [1+E7]*Z(t-1)+Y(t)$$

$$AA(t) = M(t)-Z(t)$$

$$AB(t) = -[(I11\text{^}t)*(I12\text{^}(-1+I13\text{^}t))-(I11\text{^}(t-1))*(I12\text{^}(-1+I13\text{^}(t-1)))]$$

$$AC(t) = AB(t)*X(t)$$

$$AD(t) = AB(t)*AA(t)$$

$$E5 = 100-E3$$

$$E7 = E3*E4 + E5*E6$$

We describe in greater detail the third section of part one of the spreadsheet. The specific formulas of this section are:

$$D10 = \begin{cases} D10, & \text{if } E11 > 0 \\ D11, & \text{if } E11 < 0 \end{cases}$$

$$D12 = \begin{cases} D12, & \text{if } E11 > 0 \\ D11, & \text{if } E11 < 0 \end{cases}$$

$$D11 = (D10+D12)/2$$

$$D13 = D12-D10$$

E11 = Present value of AC(t)

E13 = 0.50 (Default value--The user may alter this value)

What is important about these formulas is that they are applied iteratively (along with all the formulas in the spreadsheet) until such time as the expected present worth of the unrecovered investment is equal to zero. This is done by cutting in half the interval that contains the solution each iteration. While obtaining an expected present worth of the unrecovered investment equal to zero is the objective of the solution procedure as well as the definition of the solution, achieving zero exactly can be very time consuming. For this reason, the user is allowed to select a maximum width for the final interval (cell E13 in spreadsheet) that contains the solution. The midpoint of that interval is considered to be the solution to the problem.

When the fill rate is 1 (default value) in each year of the horizon, then this solution is the annual equivalent cost as one would find in any engineering economy text book. If the fill rate is less than 1 in any year of the horizon, then the midpoint solution gives a revenue that would be required from a completely filled investment such that the annual equivalent of all actual revenue (i.e., revenues that depend on fill rate) is equal to the annual equivalent cost.

Tables D-1 and D-2 give respectively the first and second parts of a spreadsheet for a sample problem. In table D-1, the user input values are circled and the solution is surrounded by a rectangular box. One may note in table D-2 that the values in column X and AA are identical. These columns contain the unrecovered investment calculated by two different models that clearly agree. Since the objective of the solution procedure is to make the expected net present worth of the unrecovered investment equal to zero and since the two models agree completely on unrecovered investment, then the solution given in table D-1 is correct for both models. We now proceed to the "User's Manual" that gives instructions for executing the program and its macros. (a macro is a set of programmed key strokes that may be invoked with a single key stroke.)

TABLE D-1

PART ONE OF A SAMPLE PROBLEM

Capital Structure:		Other Information: Horizon =	9
Percentage composed of debt	= 40.0%	Initial amount invested	= $1,000
Average interest rate on debt	= 9.9%	Regulatory book life	= 8
Percent composed of equity	= 60.0%	Tax life	= 5
Avg rate of return on equity	= 20.0%	Tax depreciation method	= SOYD
Composite Cost of Capital	= 15.6%	O&M growth rate	= 3.0%
Composite Income Tax Rate	= 46.0%	O&M gradient change	= $10.00
		O&M initial cost	= $50.00

Solution:			Gompertz-Makeham parameters:		
Lower Bound:	$379.00	E NPV UI:		s =	0.99
Mid Point:	$379.16	($0.3169)		g =	0.955
Upper Bound:	$379.32	Req'd Width:		c =	1.4
Width:	$ 0.318487	$0.500000			

TABLE D-2

PART TWO OF A SAMPLE PROBLEM

YEAR A	FILL RATE B	REVENUES C	SPECIAL EXPENSES D	O & M COSTS E	BOOK DEP F	ACC BOOK DEP G	TAX DEP H	DEFERRED TAXES I	ACC TAX DEFERRALS J	YEAR K
1	1	$379.16	$0.00	$ 50.00	$125.00	$ 125.00	$333.33	$ 95.83	$ 95.83	1
2	1	379.16	0.00	61.50	125.00	250.00	266.67	65.17	161.00	2
3	1	379.16	0.00	73.35	125.00	375.00	200.00	34.50	195.50	3
4	1	379.16	0.00	85.55	125.00	500.00	133.33	3.83	199.33	4
5	1	379.16	0.00	98.11	125.00	625.00	66.67	(26.83)	172.50	5
6	1	379.16	0.00	111.06	125.00	750.00	0.00	(57.50)	115.00	6
7	1	379.16	0.00	124.39	125.00	875.00	0.00	(57.50)	57.50	7
8	1	379.16	0.00	138.12	125.00	1000.00	0.00	(57.50)	0.00	8
9	1	379.16	0.00	152.26	0.00	1000.00	0.00	0.00	0.00	9

YEAR L	RATE BASE M	INTEREST ON DEBT (ACTUAL) N	INTEREST ON DEBT (ALLOWED) O	TAXABLE INCOME P	INCOME TAX Q	AFTER TAX CASH FLOW R	EQUITY INCOME (ACTUAL) S	EQUITY INCOME (ALLOWED) T	YEAR U
1	$779.17	$ 36.00	$ 36.00	($ 40.17)	($ 18.48)	$347.64	$120.00	$120.00	1
2	589.00	29.10	28.05	(21.89)	10.07	307.59	97.00	93.50	2
3	429.50	22.57	21.20	83.25	38.29	267.52	75.23	70.68	3
4	300.67	16.46	15.46	143.82	66.16	227.46	54.86	51.54	4
5	202.50	10.84	10.82	203.54	93.63	187.42	36.12	36.08	5
6	135.00	5.78	7.29	262.32	120.67	147.43	19.27	24.30	6
7	67.50	1.37	4.86	253.40	116.56	138.21	4.58	16.20	7
8	0.00	(3.39)	2.43	244.43	112.44	128.60	(11.29)	8.10	8
9	0.00	(8.55)	0.00	235.44	108.30	118.59	(28.49)	0.00	9

YEAR V	CAPITAL CONSUMPTION W	UNRECOVERED INVESTMENT X	REGULATORY SURPLUS Y	ACC FW OF SURPLUS Z	UNRECOVERD REGULATED INVESTMENT AA	PROBABILITY OF RETIREMENT IN THE YEAR AB	PROBABILITY UNRECOVERD (ACTUAL) AC	WEIGHTED INVESTMENT (REGULATED) AD	YEAR AE
1	$191.64	$808.36	($ 29.19)	($ 29.19)	$808.36	0.03	$ 22 69	$ 22.69	1
2	181.48	626.88	(4.13)	(37.88)	626.88	0.03	21.45	21.45	2
3	169.73	457.15	16.14	(27.65)	457.15	0.04	19.33	19.33	3
4	156.14	301.01	31.62	(0.34)	301.01	0.05	15.85	15.85	4
5	140.46	160.55	42.35	41.95	160.55	0.07	10.50	10.50	5
6	122.39	38.16	48.34	96.84	38.16	0.08	3.07	3.07	6
7	132.26	(94.10)	49.65	161.60	(94.10)	0.10	(9.06)	(9.06)	7
8	143.28	(237.38)	50.57	237.38	(237.38)	0.11	(26.34)	(26.34)	8
9	155.62	(393.00)	118.59	393.00	(393.00)	0.49	(192.47)	(192.47)	9

User's Manual

This user's manual begins by giving the step-by-step procedures that one would follow in order to produce the results displayed in tables D-1 and D-2. All other problems may be solved with identical steps; only the data supplied at each step will change from problem to problem. In addition, a number of other macros are available to automatically accomplish other tasks, and data items in individual cells may be changed and the problem resolved to accomplish "what if" analyses. These other macros are described after the sample problem exercise.

Sample Problem Exercise

After the Lotus 1-2-3 system has been loaded, the following steps should be followed within the Lotus environment:

1) To load and execute (retrieve) the CAPCOST model spreadsheet, type the following: /FRCAPCOST then press ENTER (carriage return). (Note: Capital letters are not required)

2) To solve a problem, one must input data using a set-up routine. To execute the set-up macro, press the (ALT) key and S at the same time. Now the program is ready for the user's input. The following message is displayed at the top of the screen:

- ENTER % COMPOSED OF DEBT (0,1):

The (0,1) means that it expects the input to be a number between 0 and 1. For example, in our problem, the percentage of the capital structure that is composed of debt is 40%. Thus, our input would be 0.40.

After each input, press ENTER. At this time the macro will automatically skip to the next cell that requires an input value. Altogether, the macro will automatically guide the user through the four sets of input data that are required.

Set 1: (Capital Structure and Income Tax Information)

The messages will read:

- ENTER % COMPOSED OF DEBT (0,1): for the example, user enters 0.40
- ENTER AVG % RATE ON DEBT (0,1): for the example, user enters 0.09
- ENTER ROR ON EQUITY (0,1): for the example, user enters 0.20
- ENTER INCOME TAX RATE (0,1): for the example, user enters 0.46

Set 2: (Time information)

The messages will read:

- ENTER TIME HORIZON: for the example, user enters 9
- ENTER AMOUNT INVESTED: for the example, user enters 1000
- ENTER REGULATORY BOOK LIFE: for the example, user enters 8
- ENTER TAX LIFE: for the example, user enters 5

As soon as the tax life is entered, the cell labeled "tax depreciation method" is highlighted and instead of a message appearing on the command line of the screen, the following menu will appear:

SOYD SL DDB/SL ACRS

The user must choose the preferred method of tax depreciation. Use left and right arrows to select the choice and then press Enter (SOYD is selected in this sample problem).

Set 3: (Operations and Maintenance Information)

The messages will read:

- ENTER O&M GROWTH RATE (0,1): for the example, user enters 0.03
- ENTER O&M GRADIENT CHANGE: for the example, user enters 10
- ENTER O&M INITIAL COST: for the example, user enters 50

Set 4: (Survival Curve Parameters)

The messages will read:

- ENTER THE s VALUE: for the example, user enters 0.99
- ENTER THE g VALUE: for the example, user enters 0.955
- ENTER THE c VALUE: for the example, user enters 1.4

The program now starts executing a completely automatic sequence of steps to find a solution to the problem. It takes between 40 and 60 seconds, on the average, to run. It will beep three times when it is done. The solution of this program consists of the revenue (column C in the spreadsheet) that will make the expected net present worth of unrecovered investment (cell E11 in the spreadsheet) equal to zero. This process is done by running an iteration program that searches for the solution that satisfies that objective. This iteration program is automatically executed at the end of the set-up macro.

Additional Activities and Macros

Suppose that the user needs to change some input cell such as time horizon, tax depreciation method, etc., after s/he has already run the set-up program. In such a case, the user need not run the set-up program again but may follow one of the following activities depending on what changes s/he has to make.

Making a Change in Time Horizon

If the user wishes to run the program again, but with a new time horizon and keeping all other inputs as before, press (ALT) and H at the same time. The following message will appear: - ENTER NEW TIME HORIZON:

Input your choice, then press ENTER. The program will then rerun the solution procedure with the new time horizon.

Making a Change in Tax Depreciation Method

If it is desired to change the tax depreciation method, (ALT) and D should be pressed at the same time. The menu described earlier will appear again on your screen. Select your choice, then press ENTER. The program will then rerun the solution procedure with the new tax depreciation method.

Making a Change in Other Input Data

If it is desired to change any input other than time horizon or tax depreciation method, simply go to the input cell you desire to change and type the new number and press ENTER. A CALC sign will appear on the lower right corner of the screen. This means that the user should press the key function F9 to recalculate the spreadsheet. In addition, press (ALT) and I at the same time to activate the iteration program. The program will then rerun the solution procedure with the changes that are made. (Note: The user may change more than one input cell before pressing (ALT) and I).

Making a Change in Fill Rate or Special Expenses

If it is desired to change the fill rate or the special expenses in any year, simply go to the desired cell in columns B or D, respectively, which corresponds to the specific year the user wants to change and type the new number and press ENTER. A CALC sign will appear on the lower right corner of the screen. This means that the user should press the key function F9 to recalculate the spreadsheet. In addition, press (ALT) and I at the same time to activate the iteration program. The program will then rerun the solution procedure with the changes that are made.

Making a Change in Probability of Retirement

If the user decides to use a different survival curve, simply go to the AB column and manually input the new probabilities in each year. The user should make sure that the sum of all the probabilities in the specified time horizon is 1. A CALC sign will appear on the lower right corner of the screen. This means that the user needs to press the key function F9 to recalculate the spreadsheet. In addition, press (ALT) and 1 at the same time to activate the iteration program. The program will then rerun the solution procedure with the new retirement probabilities.

Print

To get a printout of the spreadsheet, press (ALT) and P at the same time. Make sure that the printer is on and that the paper is at the right position on the first line of the page.

Graph

If it is desired to view some graphs on the screen, press (ALT) and G at the same time. A menu will show up on the top line of the screen with eight different options.

1) RB-T: RATE BASE VS TIME
2) SUR-T: REGULATORY SURPLUS VS TIME
3) U.INV-T: UNRECOVERED INVESTMENT VS TIME
4) SUR-U.INV: REGULATORY SURPLUS VS UNRECOVERED INVESTMENT
5) DEP-T: DEPRECIATION VS TIME (both book and tax)
6) ATCF-T: AFTER TAX CASH FLOW VS TIME
7) POR-T: PROBABILITY OF RETIREMENT VS TIME
8) QUIT: END GRAPH SESSION

Use right and left arrows to select your option and press ENTER. To go back to the menu, press ENTER again. To get out of the menu, choose the QUIT option.

Save and Quit

If it is desired to quit, press (ALT) and Q at the same time. The spreadsheet is automatically saved under the name PUCO and not under the original name CAPCOST. CAPCOST should always be an empty spreadsheet. It should be used only when desired to solve a new problem.

Summary Sheet

(ALT) S ---> Input data and run solution procedure.

(ALT) H ---> Change time horizon and run program again.

(ALT) D ---> Change tax depreciation method and run program again.

(ALT) I ---> Run solution procedure.

(ALT) P ---> Print output on printer. Make sure printer is on, and the paper is at the right position.

(ALT) G ---> View graphs on screen.

(ALT) Q ---> Save worksheet under PUCO name and exit Lotus.

* The (ALT) I (iteration program) is automatically called when the (ALT) S, the (ALT) H, or the (ALT) D are executed; no need to run (ALT) I if you are running (ALT) S, (ALT) H, or (ALT) D.

* Never save the worksheet under CAPCOST name. CAPCOST is retrieved only when solving a new problem and it should always start as an empty worksheet.